

Circuit Model of Microstrip Step Discontinuity

A.K.Verma¹, Himanshu Singh²

Department of Electronic Science, University of Delhi South Campus, N.Delhi- 110021
 Emails: 1 anandv48@bol.net.in, 2. kajal_172 @ rediffmail.com

Abstract- New closed-form models for step discontinuity shunt capacitance (C_p) and series inductance (L_s) for the microstrip step discontinuity is reported for the substrate $2.3 \leq \epsilon_r \leq 40.0$ or more. The step junction can have wide range of impedance ratio. The models have average deviation 0.027 for the C_p and 0.015 for L_s against the results of Sonnet.

Index Terms- Closed-form model, Equivalent circuit model, Microstrip step, Step discontinuity.

I. INTRODUCTION

The symmetrical step discontinuity shown in Fig.1a occurs at the junction of two microstrip lines with different characteristic impedances. It is an essential element in the design of filters, matching networks etc. It is modeled as the equivalent T-network shown in Fig. 1b. The shunt discontinuity capacitance C_p and the series discontinuity inductance L_s are determined by using the static methods [1]-[7]. Gupta and Gopinath have presented the graphical results on four types of substrates for capacitance C_p [3]-[4]. Gopinath et al. have also presented the graphical results on the inductance L_s [5]. These accurate graphical results are obtained with help of the method of moments (MOM). However, the graphical results are not suitable for the CAD purpose. Gupta et al. [6] have given closed- form expressions to compute both the shunt capacitance C_p and the series inductance L_s . These simple expressions are popular among the designers. However, these expressions are valid for a limited range of parameters with limited accuracy. Hoffman [7] has further provided some more closed- form expressions. Accuracy of these expressions is not tested against any standard method. In this work we present the

closed-form expressions to compute the step discontinuity shunt capacitance C_p and series inductance L_s . The closed-form expressions are obtained by improving the model suggested by Hoffman [7] for the capacitance C_p and the model suggested by Gupta et al. [8] for the inductance L_s . Our empirical models are obtained through the process of multidimensional logistic curve fitting. The basic data, over wide range of the parameters; are obtained from the graphical results of Gopinath and Gupta [3]-[5]. The static data are also generated from the commercial EM-software Sonnet [9] at 1 GHz. The accuracy of the present models is tested against the wide-ranging parameters obtained from these two sources. We further tested the models by incorporating the equivalent circuit model of the step discontinuity in the commercial circuit simulator- Microwave Office [10] and comparing the scattering parameters against the response obtained from the EM simulator- Sonnet [9]. The present models show acceptable results in the lower microwave frequency range. We have also compared the existing models against the results de-embedded from Sonnet.

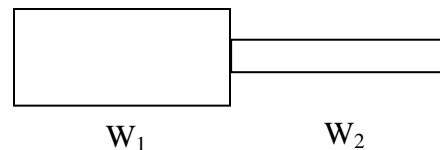


Fig.1a. Step discontinuity in microstrip

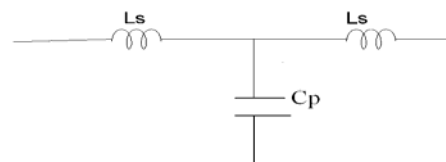


Fig.1b. Equivalent circuit of step discontinuity

II. SHUNT STEP DISCONTINUITY CAPACITANCE (C_p)

Hoffman [7] assumes that the shunt step discontinuity capacitance C_p at the junction of two microstrip lines is due to the fringe capacitance (C_f) of the wide width (W_1) line at the end. The fringe capacitance *p.u.l.* at the junction could be treated same as that of the fringe capacitance along one edge of the line. Thus the C_p is obtained as follows

$$C_p = C_f (W_1 - W_2) \quad (1)$$

Where, the fringe capacitance per unit length, C_f of the wide microstrip with substrate thickness h and relative permittivity ϵ_r is

$$C_f = \frac{1}{2} \left[\frac{\epsilon_{r,eff1}}{Z_0(\epsilon_r=1)W_0} - \frac{\epsilon_0 \epsilon_r W_1}{h} \right] \quad (2)$$

Where v_0 is velocity of the EM wave in the free-space. The effective relative permittivity $\epsilon_{r,eff1}$ of microstrip #1 and its characteristics impedance on air substrate $Z_0(\epsilon_r=1)$ are obtained from the closed-form expressions of Hammerstad and Jensen [8]. However, the shunt step discontinuity capacitance C_p computed by this method is much in error. It can be improved by incorporating an improvement factor $IF(\epsilon_r, W_1/h, W_2/h)$ that is a function of widths of both microstrip lines and relative permittivity ϵ_r of the substrate. The corrected shunt step discontinuity capacitance $C_{p(corr)}$ is given by

$$C_{p(corr)} = \left[\frac{C_p}{IF(\epsilon_r, W_1/h, W_2/h)} \right] \quad (3)$$

The improvement factor is obtained by the multidimensional logistic curve fitting of the reference data extracted from the graphical results of Gupta and Gopinath [3] and also generated from the commercial software Sonnet [9]. The data on the improvement factor, over the

wide range of parameters ϵ_r , W_1/h and W_2/h , are generated by comparing the C_p computed by equation-1 against the reference data. The power and linear regressions are used to obtain the curve-fitted expressions for the improvement factor. One variable is taken at a time and so-called constants generated in the process are really the parameters dependent on other line variables. These parameters are again curve-fitted to obtain the following set of expressions for the improvement factor.

$$IF(\epsilon_r, W_1/h, W_2/h) = 10^y$$

$$y = A (W_1/h)^{-B}$$

$$A = A_1 \left(\frac{W_2}{h} \right)^{B_1}, B = A_2 \left(\frac{W_2}{h} \right)^{B_2}$$

$$A_1 = 0.7785 + 0.0132 \epsilon_r$$

$$B_1 = 0.4023 + 0.016 \epsilon_r$$

$$A_2 = 0.632 + 0.02071 \epsilon_r$$

$$B_2 = \begin{cases} 0.2491 + 0.0077 \epsilon_r, & \text{for } 2.3 \leq \epsilon_r < 9.6 \\ 0.2491 + 0.0077 \epsilon_r, & \text{for } 9.6 \leq \epsilon_r \leq 40 \end{cases} \quad (4)$$

Fig. 2a compares the normalized step discontinuity capacitance for $\epsilon_r = 9.6$, $W_2/h = 1$ computed by present model, the method of moment results of Gupta and Gopinath, model of Hoffman and results of Sonnet. Hoffman model gives very large error. The results of Gupta and Gopinath are closer to Sonnet whereas the present model is closer to the Gupta and Gopinath. Fig.2b also demonstrates accuracy of present model for the step discontinuity on substrate $\epsilon_r = 2.3$, $W_2/h = 0.5$. Fig.3a and Fig.3b further compare present model and results of Gupta and Gopinath against the results of Sonnet for the case, $\epsilon_r = 2.3$, $W_2/h = 1$ and $\epsilon_r = 15.1$, $W_2/h = 1$. Both models show good agreement with results of Sonnet.

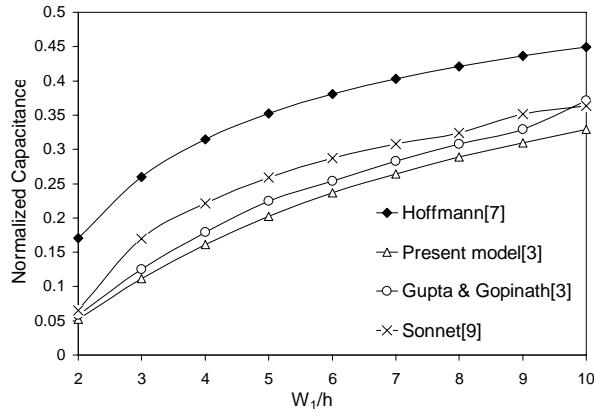


Fig.2a. Normalized step discontinuity capacitance for $\epsilon_r = 9.6, W_2/h = 1$

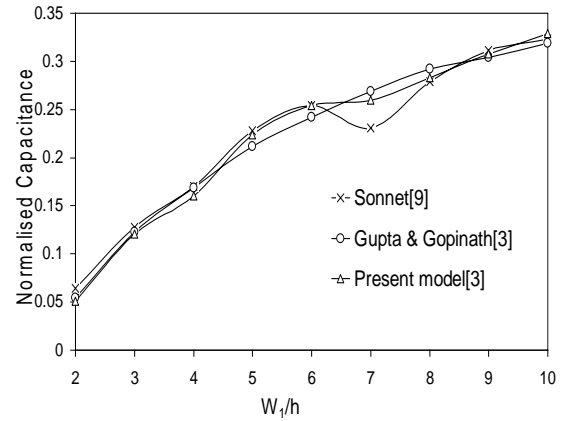


Fig.3b. Normalized step discontinuity capacitance for $\epsilon_r = 15.1, W_2/h = 1$

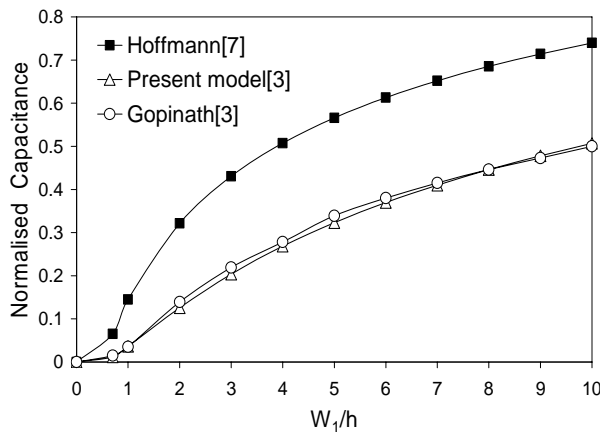


Fig.2b. Normalized step discontinuity capacitance for $\epsilon_r = 2.3, W_2/h = 0.5$

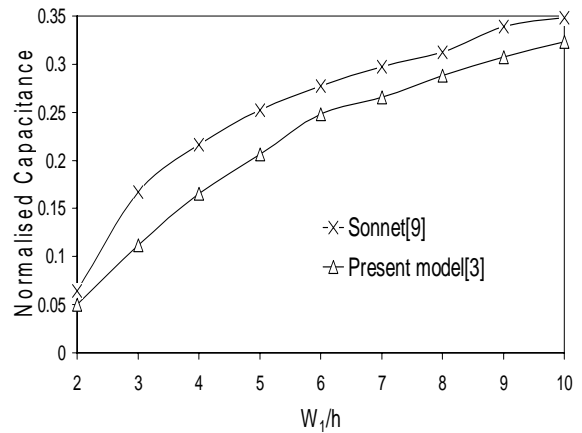


Fig.4a. Normalized step discontinuity capacitance for $\epsilon_r = 20, W_2/h = 1$

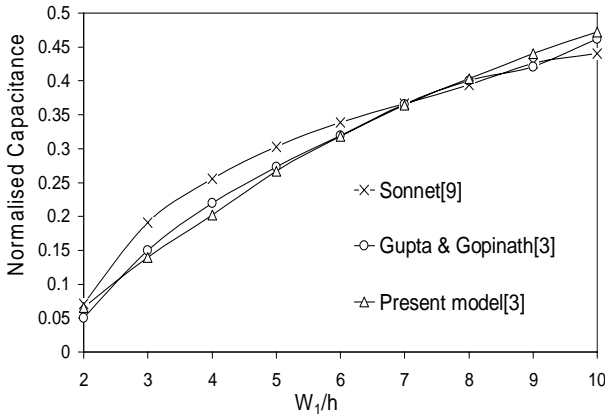


Fig.3a. Normalized step discontinuity capacitance for $\epsilon_r = 2.3, W_2/h = 1$

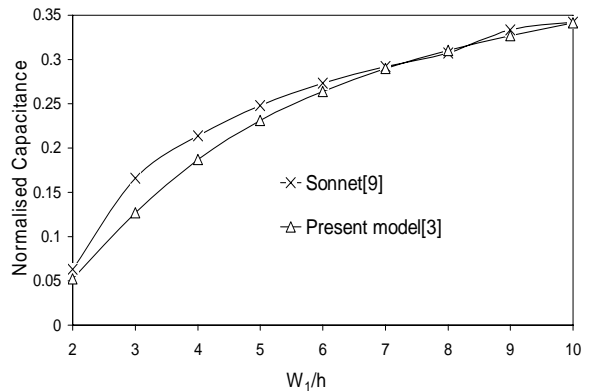


Fig.4b. Normalized step discontinuity capacitance for $\epsilon_r = 40, W_2/h = 1$

The present model is further tested for the step discontinuity on the substrate with $\epsilon_r = 20, \epsilon_r = 40, W_2/h = 1$ against the results of Sonnet. Gupta and Gopinath results are not

available for high relative permittivity substrates. The results are shown in Fig.4a and Fig.4b. It appears that present model could be useful for the microstrip step discontinuity on the substrate much higher than $\epsilon_r = 40$.

III.SERIES STEP DISCONTINUITY INDUCTANCE (L_s)

Gupta et al. [6] have provided a simple expression to compute the series step discontinuity inductance L_s that is valid for wide range of the parameters. However, the results computed from expression are at great variance from the accurate graphical results of discontinuity inductance L_s given by Gopinath et al. [5]. The results of Gopinath et al. are obtained by using the method of moments. Hoffman has also reported another expression for this purpose [7]. Schwinger and Saxon [7] have provided the closed-form expressions, summarized by Hoffman; to compute discontinuity inductance L_s of a strip line step junction. In this section we adopt the model of Gupta et al.[6] given below and use an empirically determined multiplying correction factor Y to get the improved model

$$L_S = 987 h \left(1 - \frac{Z_{02}}{Z_{01}} \sqrt{\frac{\epsilon_{r,eff2}}{\epsilon_{r,eff1}}} \right)^2 \quad (5)$$

Where, substrate thickness h is in mm. The low impedance microstrip #1 has characteristic impedance Z_{01} and effective relative permittivity $\epsilon_{r,eff1}$ and the high impedance microstrip #2 has characteristic impedance Z_{02} and effective relative permittivity $\epsilon_{r,eff2}$. The series step discontinuity inductance L_s could be normalized by the line inductance of microstrip #2

$$L_{W2} = \frac{Z_{02}(\epsilon_r = 0) \sqrt{\epsilon_{r,eff2}}}{v_0} \quad (6)$$

The characteristic impedance of microstrip on air substrate $Z_{02}(\epsilon_r = 0)$ and effective relative permittivity $\epsilon_{r,eff2}$ are computed by the closed-form expressions of Hammerstad-Jensen [8]. The

corrected step discontinuity inductance $L_{s(corr)}$ and its normalized form are given as

$$L_{s(corr)} = L_s Y \quad (a)$$

$$L_{n(corr)} = \frac{L_{s(corr)}}{L_{w2} h} \quad (b) \quad (7)$$

The expression for the multiplying correction factor Y is obtained by the multidimensional curve fitting of the data on Y generated by comparing the results of expression-5 against the reference data. The reference data are extracted from the graphical results of Gopinath et al. [5] and further data are generated from the commercial software Sonnet [9].

$$Y = Y_1 \left(\frac{w_1}{h} \right)^{-Y_2} \quad (8)$$

Where parameters Y_1 and Y_2 are given as

$$\text{For } 0.5 \leq \left(\frac{W_1}{h} \right) < 3$$

$$Y_1 = 5.3063 \left(\frac{W_2}{h} \right)^3 - 14.189 \left(\frac{W_2}{h} \right)^2 + 17.43 \left(\frac{W_2}{h} \right) - 4.551$$

$$\text{For } 3 \leq \left(\frac{W_1}{h} \right) \leq 10$$

$$Y_1 = -0.6556 \left(\frac{W_2}{h} \right)^6 + 14.832 \left(\frac{W_2}{h} \right)^5 - 107.14 \left(\frac{W_2}{h} \right)^4 + 341.36 \left(\frac{W_2}{h} \right)^3 - 512.08 \left(\frac{W_2}{h} \right)^2 + 350.29 \left(\frac{W_2}{h} \right) - 82.261 \quad (9)$$

$$\text{For } 3 \leq \left(\frac{W_1}{h} \right) \leq 10$$

$$Y_2 = -0.0485 \left(\frac{W_2}{h} \right)^6 + 0.8205 \left(\frac{W_1}{h} \right)^5 - 5.1389 \left(\frac{W_1}{h} \right)^4 + 15.196 \left(\frac{W_1}{h} \right)^3 - 22.259 \left(\frac{W_1}{h} \right)^2 + 15.582 \left(\frac{W_1}{h} \right) - 2.7651 \quad (10)$$

Fig.5a - Fig.5b compare the normalized series step discontinuity inductance L_s computed by Gupta model [6], Hoffman model [7] and present model for $W_2/h = 0.5$ and 2 respectively on the substrate with $\epsilon_r = 9.6$. The step discontinuity inductance is independent of relative permittivity. The MOM based results from Gopinath et al. [5] are taken as reference. It is apparent that Gupta model and Hoffman model are useful only for a limited range of parameters. The present model shows good agreement over the wide range of parameters. Fig.6 also compares the results obtained from Schwinger and Saxon [7] model. As this model is developed for the series step discontinuity inductance in the strip line, so we have taken only one half of the value obtained from the model for the microstrip case. Finally Fig.7 compares all these model against the results obtained from the commercial software Sonnet. The present model shows very good agreement for W_1/h up to 10.

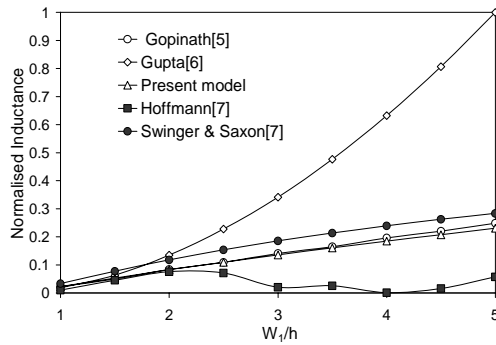


Fig.5a. Normalized series step discontinuity inductance L_s for $\epsilon_r = 9.6$, $W_2/h = 0.5$

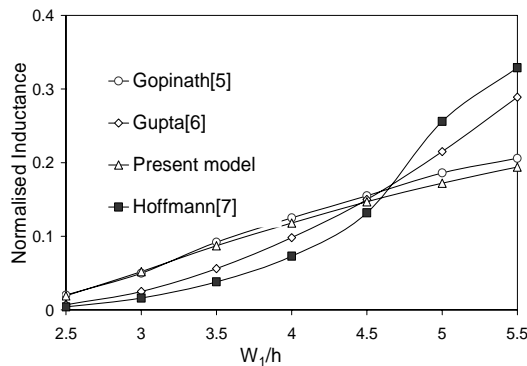


Fig.5b. Normalized series step discontinuity inductance L_s for $\epsilon_r = 9.6$, $W_2/h = 2.0$

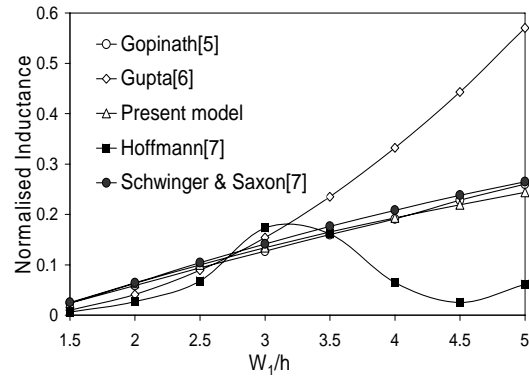


Fig.6. Normalized series step discontinuity inductance L_s for $\epsilon_r = 9.6$, $W_2/h = 1.0$

IV. SCATTERING PARAMETER OF DISCONTINUITY CIRCUIT MODEL

We can simulate the circuit model shown in Fig.1b in the Microwave Office [10] after calculating the component values from our present closed-form models. The microstrip step structure can be also be constructed in the Sonnet [9]. The scattering parameters S_{11} , S_{12} , S_{22} and S_{21} are obtained from both the circuit simulator- Microwave Office and the EM-simulator- Sonnet to compare accuracy of the present discontinuity models at the response performance level. Fig.8 shows such comparison of S_{11} and S_{21} for $\epsilon_r = 9.6$, $W_2/h = 1.0$, $W_1/h = 5.0$. As the present model is a frequency independent static model, it has acceptable performance up to 10 GHz. The static model needs improvement for its operation in high frequency range.

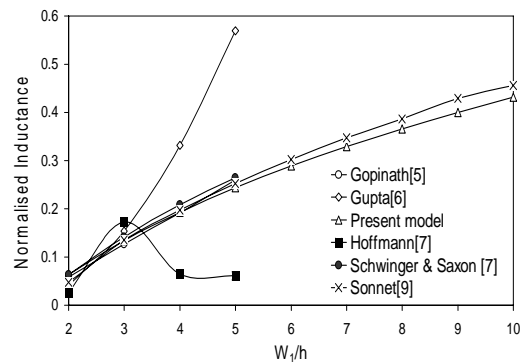


Fig.7. Comparison of step inductance models against Sonnet for $\epsilon_r = 9.6$, $W_2/h = 1.0$

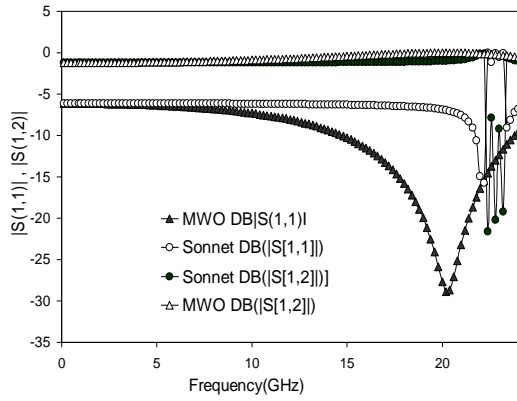


Fig.8. Comparison of circuit model against Sonnet for $\epsilon_r = 9.6$, $W_2/h = 1.0$, $W_1/h = 5.0$

V. CONCLUSION

We have presented the design oriented new closed-form expression for the microstrip step discontinuity. The models of the step discontinuity shunt capacitance and series inductance are valid for the wide range of physical parameters. We have tested accuracy of the models against the commercial software Sonnet and MOM based results of Gopinath. Typical comparisons of the present models and MOM results against the results obtained from Sonnet are shown in Table-1 and Table -2. In case of the shunt capacitance C_p the results from both the MOM model and present closed-form models

are almost identical for $\epsilon_r = 9.6$. The present model has average deviation 0.04 and MOM has average deviation 0.043 from the results of Sonnet. With increase in relative permittivity the average deviation of the present model decreases as for $\epsilon_r = 20.0$ it is 0.031 and for $\epsilon_r = 40.0$ it is 0.011. Thus the model can provide accurate results for much higher relative permittivity substrate. The deviation is less for larger impedance ratio step. In case of the series inductance the average deviation of the present model is 0.015.

Table 1: Normalized discontinuity inductance L_s for $W_2/h = 1.0$, $\epsilon_r = 9.6$

W_1/h	Sonnet	Gopinath	Difference	Present Model	Difference
2.0	0.048	0.049	0.001	0.063	0.015
3.0	0.135	0.127	0.008	0.134	0.001
4.0	0.197	0.191	0.006	0.193	0.004
5.0	0.253	0.260	0.007	0.264	0.011
6.0	0.303	-	-	0.289	0.014
7.0	0.347	-	-	0.329	0.018
8.0	0.386	-	-	0.366	0.020
9.0	0.429	-	-	0.400	0.029
10.0	0.456	-	-	0.432	0.042

Table 2: Normalized discontinuity Capacitance C_p

$W_2/h=1.0$	$\epsilon_r = 9.6$					$\epsilon_r = 20.0$			$\epsilon_r = 40.0$		
	W_1/h	Sonnet	Gopinath	Difference	Present Model	Difference	Sonnet	Present Model	Difference	Sonnet	Present Model
2.0	0.065	0.050	0.015	0.052	0.013	0.064	0.050	0.014	0.063	0.052	0.011
3.0	0.171	0.128	0.043	0.111	0.060	0.167	0.112	0.055	0.166	0.127	0.039
4.0	0.223	0.162	0.061	0.161	0.062	0.216	0.165	0.051	0.214	0.187	0.027
5.0	0.259	0.200	0.059	0.202	0.057	0.252	0.206	0.046	0.248	0.231	0.017
6.0	0.287	0.231	0.056	0.232	0.055	0.277	0.248	0.029	0.273	0.264	0.009
7.0	0.308	0.250	0.058	0.264	0.044	0.297	0.266	0.031	0.292	0.294	0.002
8.0	0.324	0.278	0.046	0.289	0.035	0.312	0.288	0.024	0.307	0.310	0.003
9.0	0.352	0.308	0.044	0.310	0.042	0.339	0.307	0.032	0.333	0.327	0.006
10.0	0.361	0.318	0.043	0.329	0.032	0.348	0.323	0.025	0.342	0.341	0.001

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