RF Analysis of Nano Transmission Lines using Meshless Method

S. Kanthamani*, Dr. (Mrs.). S. Raju, Dr.V. Abhaikumar, Dr.V.Mohan

RF Systems Lab, Department of ECE
Thiagarajar College of Engg. Madurai-620515, TN, INDIA
Tel: 0452-2482240; Fax: 0452-2484327; E-mail: skmece@tce.edu,skanthamani@yahoo.ca

Abstract—Quasi-static analysis of nano transmission lines using collocation method is presented. Collocation approach – a mesh less technique is more efficient in computation as compared with other regular discretisation approaches due to its adaptive nature. Numerical results for characteristic impedances (Zo) of nano transmission lines obtained using quasi-static capacitance are presented. Validation is done by comparing the obtained results with Littuinger liquid theory model and good agreement is obtained. The electromagnetic simulation of the lumped element model of the carbon nano tube is done using ADS.

Index terms—Nano transmission line, meshless method, Littuinger, carbon nano tube.

I. INTRODUCTION

Nano technology is an emerging interdisciplinary area that focuses on the structures of nanoscopic dimensions. Carbon nanotubes (CNTs) are the most studied class of nanotube/nanowire. The realistic sizes of integrated circuits approaching the 10nm length scale has motivated research in the electronic properties of nano-scale devices [1]. A CNT consists of a sheet of graphite rolled up into a tube. They can be single-walled or multi-walled and be either metallic or semi-conducting [2]. Because of high current densities and increased reliability, metallic Single-walled Carbon Nanotubes (SWCNTs) have been subjected to fundamental research both in theory as well as experiments [3]. One of the potential applications of CNT is as a transmission line at RF and microwave frequencies [4].

By modeling the CNT as a nano transmission line with distributed kinetic and magnetic inductances as well as distributed quantum capacitance and electrostatic capacitance it has been reported that the complex frequency dependent impedance for a variety of geometries can be calculated [5].

The mechanical behavior of single walled carbon nanotubes has been studied by Tserpes k.i and Papanikos P [6] using finite element method (FEM). This method involves mesh generation, mesh compatibility, re-meshing and interpolation of solutions between the domains for solving the governing partial differential equations of the chosen problem. In miniaturized geometries such as CNT mesh generation is difficult. Further it is time consuming to perform the self-consistent analysis of the mixed energy domains governing the behavior of the device.

To overcome these difficulties an efficient approach such as mesh-less or mesh-free method has been proposed for numerical solution of partial differential equations corresponding to two or three-dimensional structures [7]. Mesh less techniques requires only a scattered set of nodes representing the domain of interest. No connectivity information among scattered set of nodes is required, unlike finite element, boundary element or finite difference techniques.

Several mesh-less methods such as classical smooth particle hydrodynamics approach, generalized finite difference method, diffuse element method, finite point method, element free Galerkin method have been proposed in the literature [8]. In many of these methods the
Interpolation functions are constructed by employing either the reproducing kernel method or moving least square method. Once the interpolation functions are constructed, the governing partial differential equations can be solved employing either a Galerkin or a collocation approach. Galerkin-based approaches are typically not true meshless technique as they require some kind of a background grid for integration purpose. A collocation approach on the other hand, can be a true meshless technique and does not require any background grid [9].

In this paper the quasi-static capacitance and the characteristic impedance of a nano transmission line is calculated using collocation method.

II. PROBLEM STATEMENT

The objective of this paper is to obtain the quasi static electrical parameter of nano transmission line namely the characteristic impedance and also find the radio frequency performance. The geometry of the Nano Transmission line chosen for analysis is shown in Fig. 1.

A. Mathematical Formulation

The governing partial differential equations in the domain of interest are given as

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \]  

where \( \phi \) is the potential distribution.

The boundary conditions are

**Region 1:**
(i) \( \phi = 0 \) at \( x = 0 \) & \( 0 < y < e \)  
(metallic enclosure)
(ii) \( \phi = 0 \) at \( y = 0 \) & \( 0 < x < a \)  
(metallic enclosure)
(iii) \( \frac{\partial \phi_1}{\partial y} = \frac{\partial \phi_6}{\partial y} \) at \( y = e \)  
(interface between dielectric and substrate)

**Region 2:**
(iv) \( \phi = 0 \) at \( a < x < b \)  
(metallic enclosure)
(v) \( \phi = 1 \) at \( y = e - \sqrt{2cx - g^2 - x^2 - c^2} \)  
(on nano tube)
Region 3:
(vi) \( \varphi = 0 \) at \( y = 0 \) & \( b < x < d \) 
(metallic enclosure)

(vii) \( \varphi = 0 \) at \( x = 0 \) & \( d < y < e \) 
(metallic enclosure)

(viii) \( \frac{\partial \varphi_3}{\partial y} = \frac{\partial \varphi_4}{\partial y} \) at \( y = e \) 
(interface between dielectric and substrate) \( (4) \)

Region 4:
(ix) \( \varphi = 0 \) at \( y = f \) & \( b < x < d \) 
(metallic enclosure)

(x) \( \varphi = 0 \) at \( x = d \) & \( e < y < f \) 
(metallic enclosure)

(xi) \( \frac{\partial \varphi_3}{\partial y} = \frac{\partial \varphi_4}{\partial y} \) at \( y = e \) 
(interface between dielectric and substrate) \( (5) \)

Region 5:
(xii) \( \varphi = 0 \) at \( a < x < b \) 
(metallic enclosure)

(xiii) \( \varphi = 1 \) at \( y = e + \sqrt{2cx - g^2 - c^2 - x^2} \) 
(on nano tube) \( (6) \)

Region 6:
(xiv) \( \varphi = 0 \) at \( x = 0 \) & \( e < y < f \) 
(metallic enclosure)

(xv) \( \varphi = 0 \) at \( y = f \) & \( 0 < x < a \) 
(metallic enclosure)

(xvi) \( \frac{\partial \varphi_3}{\partial y} = \frac{\partial \varphi_6}{\partial y} \) at \( y = e \) & \( 0 < x < a \) 
(interface between dielectric and substrate) \( (7) \)

The problem now is to solve the two-dimensional Laplace equation using mesh less method to find the potential distribution and hence the electrostatic capacitance and the characteristic impedance of the nano transmission line.

B. Collocation Approach

Collocation method approximates solution to boundary value problem by finite linear combination of basis functions.

For two – point boundary value problem
\[ u'' = f(t,u,u'), a < t < b \]
with boundary condition
\[ u(a) = \alpha, u(b) = \beta \]
An approximate solution of form
\[ u(t) \approx v(t,x) = \sum_{i=1}^{n} x_i \varphi_i(t) \]
where \( \varphi_i \) are basis functions defined on \([a,b]\) and \( x_i \) is \( n \)-vector of parameters to be determined.[10].

The same method can also be extended to solve the partial differential equation involved for the problem chosen.

C. Collocation formulation

The general solution of the problem space is expressed as polynomials in terms of arbitrary constants
\[ \varphi = A + Bx + Cy + Dxy \]

Approximate solution of the CNT is obtained by solving equation (1) using collocation method after the application of the boundary conditions are

Region 1:
\[ \varphi_1 = x(y(A_1 + B_1x + C_1y + D_1xy)) \]
Region 2:
\[ \varphi_2 = y(A_2 + B_2x + C_2y + D_2xy) \]
Region 3:
\[ \varphi_3 = x(y - d)(A_3 + B_3x + C_3y + D_3xy) \]
Region 4:
\[ \varphi_4 = (x - d)(y - f)(A_4 + B_4x + C_4y + D_4xy) \]
Region 5:
\[ \varphi_5 = y(A_5 + B_5x + C_5y + D_5xy) \]
Region 6:
\[ \varphi_6 = x(y - d)(A_6 + B_6x + C_6y + D_6xy) \]

(11)

where \( A_i - A_6, B_1 - B_6, C_1 - C_6, D_1 - D_6 \) are the arbitrary constants to be determined through collocation method.

The resulting solution obtained through the proposed point collocation method for the entire structure is given as

\[ \varphi_1 = [x(A_1y) + (B_1xy)] \]
\[ \varphi_2 = \left[ \frac{y}{10 - \sqrt{20x + 1 - x^2 - 100}} \right] \]
\[ + C_2 \left( y^2 - y \left( 10 - \sqrt{20x + 1 - x^2 - 100} \right) \right) \]
\[ + D_2 \left( y_2x - xy \left( 10 - \sqrt{20 - x^2 + 1 - 100} \right) \right) \]
\[ \varphi_3 = [y(x - d)(A_3 + B_3x)] \]
\[ \varphi_4 = [(y - f)(x - d)(A_4 + B_4x)] \]
\[ \varphi_5 = \left[ C_5 \left( y^2 - y \left( 10 + \sqrt{20 + 1 - x^2 - 100} \right) \right) \right] \]
\[ + D_5 \left( y^2x - xy \left( 10 + \sqrt{20 + 1 - x^2 - 100} \right) \right) \]
\[ \varphi_6 = [(y - f)(A_6x) + B_6x] \]

(12)

III. NUMERICAL RESULTS AND DISCUSSIONS

Matlab coding has been developed to solve the potential functions (\( \Phi \)) at each chosen point.

Random distributions of 3482 points are considered. Using the standard formulas available in [4] the distributed kinetic inductance (\( L_M \)), magnetic inductance (\( L_K \)) as well as distributed quantum capacitance (\( C_Q \)), of the nano transmission line is evaluated. The electrostatic capacitance (\( C_E \)) and the characteristic impedance (\( Z_T \)) obtained by proposed method for a change in the radius of nanotube has been compared with the Littuinger theory model results. The results of the present work are in good agreement with the Littuinger liquid theory model as given in Table 1.

<table>
<thead>
<tr>
<th>Radius in (nm)</th>
<th>( C_E ) (aF/( \mu )m)</th>
<th>( Z_T ) (k ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r=1 )</td>
<td>50</td>
<td>49</td>
</tr>
<tr>
<td>( r=2 )</td>
<td>58</td>
<td>55</td>
</tr>
<tr>
<td>( r=3 )</td>
<td>64</td>
<td>60</td>
</tr>
<tr>
<td>( r=4 )</td>
<td>69</td>
<td>64</td>
</tr>
<tr>
<td>( r=5 )</td>
<td>73</td>
<td>68</td>
</tr>
</tbody>
</table>

Table 1: Comparison of \( C_E \) and \( Z_T \) between Littuinger theory and present work

Fig 3 and 4 shows the variation of electrostatic capacitance and the characteristic impedance for change in the radius of the carbon nano tubes for the proposed method and method proposed by P.J.Bruke.
Fig 4. Characteristic impedance ($Z_T$) for different radius (r)

The minor discrepancies of about one percent error in the Electrostatics capacitance obtained through mesh less method, with respect to Littuinger theory model can be reduced further by using a higher order polynomial approximation for the unknown solution.

A. RF performance of carbon nano tube

The radio frequency performance of the carbon nano tube is also obtained using the commercial electromagnetic simulator Agilent ADS. The performance is carried out using the proposed equivalent circuit model based on Littuinger liquid theory model as shown in Fig 5. The values of the equivalent circuit components are found using the formulas available [4]. Moreover RF performance is verified using the electrostatic capacitance obtained using the proposed meshless method.

![Equivalent circuit diagram for an SWNT](image)

Kinetic Inductance:

The Kinetic inductance per unit length is given as [3]

$$L_{kinetic} = \frac{\hbar}{2e^2V_F}$$  \hspace{1cm} (14)

The Fermi velocity for graphene and also carbon nanotubes is usually taken as $V_F = 8 \times 10^5$ m/s, so that numerically $L_K = 16 \text{nH}/\mu\text{m}$

Electrostatic capacitance:

The electrostatic capacitance between the a wire and a ground plane is given as [3]

$$C_E = \frac{2\pi\varepsilon}{\cosh^{-1}\left(\frac{2h}{d}\right)}$$  \hspace{1cm} (15)

Quantum capacitance:

The Quantum capacitance per unit length is given as

$$C_Q = \frac{2e^2}{hV_F}$$  \hspace{1cm} (16)

where d is the nano tube diameter and h is the distance to the ground plane, $\varepsilon$ is the dielectric constant.

Characteristic impedance:

The characteristic impedance is given as

$$Z_T = \sqrt{\frac{L_{Total}}{C_{Total}}}$$

$$= \sqrt{(L_K + L_M) \left( \frac{1}{C_Q} + \frac{1}{C_{ES}} \right)}$$  \hspace{1cm} (17)

Figure 6(a) shows the schematic layout of the equivalent circuit model proposed by P.J.Bruke performed using Agilent ADS. The return loss and the transmission coefficients are nearly -28 dB and -0.002 dB in the 0.1 to 20 GHz frequency range as available in figure 6(b).
The RF performance characteristics of the nano transmission line have been improved by placing the tubes in parallel as shown in figure 7(a). The simulation results in a return loss of -10 dB at 100 GHz as shown in figure 7(b).
Fig. 7 (b): Simulated magnitude of $S_{11}$ and $S_{12}$ of CNTs arranged in parallel with $L_{H}=1$ (pH/μm), $L_K=16$ (pH/μm), $C_Q=100$ (aF/μm) over the frequency range (0.1 to 100GHz)

Figure 8 shows the compared results of variations of return loss of the nano transmission lines with different electrostatic capacitances($C_E$) obtained using Littuinger liquid and collocation approach.

IV. CONCLUSION

A novel mesh – less analysis of carbon nano tube transmission line based on collocation approach is presented. The approach is validated by comparing the results with Littuinger liquid theory. The simulation results show that the possibility of CNT application in wireless designs. Research work is in progress to model the nano transmission line as filters operating at microwave frequencies.

ACKNOWLEDGMENT

This work was supported by TIFAC CORE in Wireless Technologies, Thiagarajar Advanced Research Center, Thiagarajar College of Engineering, Madurai, Tamil Nadu, India.

REFERENCES