Influence of SOA-Induced Chirp on PMD Tolerance in 10Gbps Transmission Systems Using SOAs as Booster Amplifiers

Hodeok Jang*, Yonghoon Kim, Sub Hur, Seungki Nam, and Jichai Jeong
Department of Radio Engineering, Korea University, 1, 5Ka, Anam-dong, Sungbuk-ku, Seoul, 136-701, Korea
Tel: 82-2-3290-3233, Fax: 82-2-924-9710, E-mail: teri20@korea.ac.kr

Abstract—We have investigated the polarization-mode dispersion (PMD) tolerance degraded by semiconductor optical amplifier (SOA)-induced chirp for 10 Gb/s nonreturn-to-zero (NRZ), duobinary NRZ, return-to-zero (RZ), and carrier-suppressed RZ (CS-RZ) modulation formats.

Index Terms—polarization mode dispersion, modulation format, semiconductor optical amplifier

1. Introduction
Polarization mode dispersion (PMD) is one of the critical hurdles for next-generation high-bit-rates and long-haul optical transmission systems [1], [2]. Due to the fiber birefringence, a single mode fiber (SMF) supports two orthogonal polarization modes traveling at different velocities along the fiber. The difference in traveling time between the two principal states of polarization (PSP) is called as differential group delay (DGD). Also, PMD has random and time-varying characteristics since the fiber birefringence changes along a fiber. It becomes a dominant source of pulse distortion in dispersion managed high bit rate transmission systems. Specifically, high PMD values are of special interest since a fair amount of the installed fibers has PMD values 10–100 times larger than that of the current state-of-the-art.

The PMD tolerance has been investigated for various modulation formats. For the first-order PMD only, the duobinary modulation format has the worst tolerance since the signal with larger duty cycle tends to have larger inter-symbol interference (ISI) [3]. The return-to-zero (RZ) modulation format has the best tolerance against the first-order PMD due to its small duty cycle. On the other hand, RZ-shaped signals are usually less tolerant to the second-order PMD than nonreturn-to-zero (NRZ) signals due to their wide optical spectra. The duobinary modulation format is superior to the NRZ modulation format in tolerance to the first- and second-order PMD due to its narrower power spectrum, whereas the NRZ modulation format is better tolerance to the first-order PMD than the duobinary modulation format [4]. These results have been obtained under the condition which a transmission link has only the PMD effect. For the presence of dispersion and nonlinear effects occurred in a transmission link, therefore, it is necessary to investigate the PMD tolerance in details.

In this paper, we investigated the first- and second-order PMD tolerance due to NRZ, duobinary NRZ, RZ, and CS-RZ modulation formats in 10 Gb/s transmission systems with semiconductor optical amplifiers (SOAs) as booster amplifiers. The modulation formats play an important role in the transmission performance of high speed systems with and without PMD compensation. A modulation format more resilient to the PMD effect guarantees improved receiver sensitivity and increased transmission length. Meantime, SOAs as booster amplifiers have the advantages of not only the increase of optical power into the fiber but also the conversion of the positive chirp into the negative chirp [5]. In the 1550 nm wavelength region, the self-phase modulation (SPM) induced by high launching power into the fiber and the negative chirp helps the transmission performance for high-bit-rate transmissions through SMF to be improved. In this paper, we analyze the PMD tolerance of signals with the SOA-induced chirp, in terms of
the receiver sensitivity at $10^{-9}$ BER degraded by a given DGD for optically preamplified receivers. Also, for a specific DGD with the same value as half a bit duration, we investigated the influence of SOA-induced chirp on the PMD tolerance.

The paper is organized as follows. In section 2, we present our modeling method of PMD, SOAs, and optical receivers. Section 3 and Section 4 are dedicated to an analysis on PMD tolerance of modulation formats with SOA-induced chirp. In detail, we analyze the influence of the SOA-induced chirp on the tolerance to the first-order PMD only as well as the first- and second-order PMD. Finally, in section 5 we summarize our results and discuss on modulation formats with PMD tolerance resilient to SOA-induced chirp.


2.1 SOAs Model

![Fig.1 Schematic diagram of illustrating wave propagation on large-signal dynamic SOA model based on the time-dependent TMM](image)

SOAs was modeled using the time-dependent transfer matrix method (TMM). Since the conventional TMM cannot estimate the dynamic characteristics of the semiconductor optical devices, the time-dependent TMM was employed for simulating the dynamic characteristics of SOAs. Fig. 1 shows the schematic of illustrating the wave propagation on a large-signal dynamic SOA model based on the time-dependent TMM. The time-dependent TMM was used for solving the pulse propagation equation in the device. In every section, the complex pulse envelope is calculated by self-consistently solving the pulse propagation equations, the gain equation, and the rate equation for the carrier density. The amplified spontaneous emission (ASE) noise is calculated from the gain equation and the spontaneous emission factor [6].

2.2 Wave Propagation in Optical Fibers in Presence of PMD

The PMD effect in the fiber can be modeled as a concatenation of several birefringent segments. The input field of birefringent segments is represented by $E_{in}(t) = E_{in} \cdot j_{in}$, where $j_{in}$ is the input Jones polarization vector which depends on the azimuth and the ellipticity of the input SOP [7]. The output field is given by [8]

$$E_{out}(\omega) = T(\omega) \cdot E_{in}(\omega) \cdot j_{in} = E_{out}(\omega) \cdot j_{out}(\omega)$$  (1)

where $T(\omega)$ represents the transfer matrix of fiber and $j_{out}(\omega)$ is given by a matrix multiplication of the unitary matrix $U(\omega)$ and the input Jones polarization vector $j_{in}$. $E_{out}(\omega)$ is obtained by the Fourier transformation of output signals of a lossy and dispersive fiber with nonlinear effects.

In Eq. (1), the unitary matrix $U(\omega)$ can be written as $U(\omega) = R^{-1}(\omega)D(\omega)R(\omega)$, where $R(\omega)$ takes into account the rotation of PSPs, and the dispersive matrix $D(\omega)$ takes into account the different propagation speeds on the two PSPs [7], [8].

$$R(\omega) = \begin{bmatrix} \cos k\omega & \sin k\omega \\ -\sin k\omega & \cos k\omega \end{bmatrix}$$  (2)

$$D(\omega) = \begin{bmatrix} \exp[j\Delta\tau\omega/2] & 0 \\ 0 & \exp[-j\Delta\tau\omega/2] \end{bmatrix}$$  (3)

The PMD dispersion vector represented by the second-order approximation is given by $\Omega(\omega) = (\Delta\tau + \omega \cdot \Delta\tau_{a}) \cdot \vec{q} + \Delta\tau \cdot \omega \cdot 2k$, where $\Delta\tau_{a} = \partial\Delta\tau / \partial\omega$ and $2k = \partial\vec{q} / \partial\omega$ [9], [10]. The second-order PMD effects are represented by frequency dependent terms; the linear DGD frequency dependence ($\Delta\tau_{a}$) and the PSP rotation with a constant angular rate ($2k$). Here, $2k$ perpendicular to $\vec{q}$ causes depolarization [9], [10], [11]. This depolarization is a result of the rotation of PSPs with frequency. The linear DGD frequency dependence induces the polarization dependent chromatic dispersion (PDC). The PDC eliminates the chromatic dispersion in the condition of the first-order PMD compensation and small PSP rotation. In the second-order PMD approximation, the time domain output field obtained by inverting Eq. (1)
can be represented as [8]

\[
\begin{align*}
\hat{E}_{\text{out}} &= \frac{1}{2\sqrt{2}} \left( a u^* + b u \right) \left( E_{\text{out}}(t + \Delta\tau/2) + E_{\text{out}}(t - \Delta\tau/2) \right) \\
&+ a u \left( E_{\text{out}}(t - 2k + \Delta\tau/2) - E_{\text{out}}(t - 2k - \Delta\tau/2) \right) \\
&+ b u \left( E_{\text{out}}(t + 2k + \Delta\tau/2) - E_{\text{out}}(t + 2k - \Delta\tau/2) \right)
\end{align*}
\]

(4)

where \( E_{\text{out}}^+(t) \) is the time domain output field with considering loss, chromatic dispersion, and polarization-dependent chromatic dispersion in fiber, \( a = e^{i\theta} \cos(\epsilon + \pi/4) \), \( b = e^{i\theta} \cos(\epsilon - \pi/4) \), where \( \theta \) is an azimuth and \( \epsilon \) is an ellipticity of the input SOP, \( u = [1 \ j] \) and \( u^* = [1 - j] \).

2.3 Receiver Model

The receiver model consists of optical amplifier, followed by a Gaussian band-pass optical filter, a square law detector, and a fifth-order Bessel-Thomson electrical filter [12]. The bandwidth of electrical and optical filters of receivers was optimized for various modulation formats, as shown in Table 1. The shot noises, beat noises generated by signal and amplified spontaneous emission (ASE) noise, and the receiver circuit noises were considered in calculation of bit error rate (BER), which was optimized with both threshold level and sampling time [13]. The receiver sensitivity at \( 10^{-9} \) BER was used to measure the PMD-induced penalty.

<table>
<thead>
<tr>
<th>Modulation format</th>
<th>Electrical bandwidth (GHz)</th>
<th>Optical bandwidth (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRZ</td>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>RZ</td>
<td>6</td>
<td>50</td>
</tr>
<tr>
<td>Duobinary</td>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>CS RZ</td>
<td>6</td>
<td>50</td>
</tr>
</tbody>
</table>

3. First-Order PMD Tolerance due to SOA-Induced Chirp

Fig. 2 shows the schematic diagram of a transmission system. Transmitters for various modulation formats generate 10 Gb/s PRBS signals with \( 2^7 \) bit. SOA was used for amplifying the input signals launched into an optical fiber. In the fiber, dispersion, loss, and fiber nonlinearities was neglected to consider only the PMD effect. To analyze the first-order PMD tolerance of NRZ, RZ, duobinary NRZ, and CS-RZ modulation formats, we neglected \( \Delta\tau_p \) and \( |2k| \) representing the second-order PMD.

Fig.3 shows the first-order PMD-induced power penalty at \( 10^{-9} \) BER for a given SOA-induced chirp with the incident angle of 45°. The incident angle is an angle between the SOP of input signals and the PSPs of the fiber. When the incident angle is 45°, the deformation of signals by DGD becomes the largest since the inter-symbol interference (ISI) between the PSPs of the fiber is maximized. As shown in Fig.3, all
modulation formats have very similar PMD-induced power penalty regardless of the peak-to-peak value of the chirp induced by SOAs. That is, the first-order PMD tolerance is hardly affected by the SOA-induced chirp.

4. First- and Second-Order PMD Tolerance due to SOA-Induced Chirp

To analyze the tolerance of the first- and second-order PMD for modulation formats, we set $\Delta \tau_\omega$ and $2k$ representing the second-order PMD to their rms values for a given DGD. The rms values of $\Delta \tau_\omega$ and $2k$ are decided by theoretical scaling rule based on measurements from several fibers with different mean DGDs and a statistically significant amount of data for each fiber [8]. Table 2 shows the rms value of $\Delta \tau_\omega$ and $2k$ for a given DGD.

<table>
<thead>
<tr>
<th>$\Delta \tau$ (ps)</th>
<th>$\Delta \tau_\omega$ (ps/GHz)</th>
<th>$2k$ (°/GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.02267</td>
<td>0.4147</td>
</tr>
<tr>
<td>20</td>
<td>0.09068</td>
<td>0.8294</td>
</tr>
<tr>
<td>30</td>
<td>0.20403</td>
<td>1.2441</td>
</tr>
<tr>
<td>40</td>
<td>0.36272</td>
<td>1.6588</td>
</tr>
<tr>
<td>50</td>
<td>0.56675</td>
<td>2.0735</td>
</tr>
<tr>
<td>60</td>
<td>0.81612</td>
<td>2.4882</td>
</tr>
<tr>
<td>70</td>
<td>1.11083</td>
<td>2.9029</td>
</tr>
</tbody>
</table>

Fig. 4 shows the power penalties due to the first- and second-order PMD at $10^{-9}$ BER for a given SOA-induced chirp. The PMD-induced power penalty of all modulation formats except for the CS-RZ modulation format increases as the peak-to-peak value of the SOA-induced chirp increases from 0 Å to 0.28 Å. Among all the modulation formats considered here, the NRZ modulation format has the most sensitive PMD tolerance to the SOA-induced chirp. With DGD of 50 ps, $\Delta \tau_\omega$ of 0.57 ps/GHz, and $2k$ of 2.07 °/GHz, NRZ signals with the peak-to-peak chirp of 0.28 Å have 3.9 dB larger PMD-induced power penalty than that without SOAs as booster amplifiers. For the RZ and duobinary NRZ modulation formats, they have 1.5 dB and 2.4 dB larger PMD-induced power penalty than those without SOAs as booster amplifiers, respectively. However, the CS-RZ modulation format has almost constant PMD tolerance regardless of the SOA-induced chirp. In other words, its PMD tolerance is nearly insensitive to the chirp induced by SOAs.

Fig. 5 shows the receiver sensitivities at $10^{-9}$ BER and the power penalties induced by the PMD according to the peak-to-peak value of the SOA-induced chirp. We calculated the power penalties for a fixed PMD condition; DGD = 50 ps which is the same value as half a bit duration, $\Delta \tau_\omega = 0.57$ ps/GHz, and $2k = 2.07$ °/GHz. For the NRZ modulation format, its PMD tolerance is deteriorated from 2.5 dB to 14.1 dB as the peak-to-peak value of the SOA-induced chirp increases from 0 Å to 0.6 Å. The PMD tolerance becomes worse by 3 dB when the signal has the peak-to-peak chirp of about 0.24 Å. For the duobinary NRZ modulation format, its PMD tolerance is deteriorated from 1.3 dB to 15.2 dB as the peak-to-peak value of the SOA-induced chirp increases from 0 Å to 0.6 Å. Especially, above 0.3 Å peak-to-peak chirp, the PMD-induced power penalties abruptly increase. The PMD tolerance becomes worse by 3 dB when the signal has the peak-to-peak chirp of about 0.31 Å. For the RZ modulation format, its PMD tolerance is gradually deteriorated from 2.5 dB to 5.5 dB as the peak-to-peak value of the SOA-induced chirp increases from 0 Å to 0.6 Å. The value of peak-to-peak chirp which deteriorates the PMD tolerance by 3 dB increases up to 0.6 Å.
However, for the CS-RZ modulation format, it has almost no change in PMD-induced penalties up to 0.6 Å. Consequently, the chirp induced by SOAs affects the PMD tolerance of the NRZ, RZ, and duobinary NRZ modulation formats except for the CS-RZ modulation format. Specifically, the value of peak-to-peak chirp generating a 3dB additional power penalty increases in the following order: NRZ, duobinary NRZ, and RZ modulation formats. So, in terms of the 3dB additional power penalty, the first- and second-order PMD tolerance of the NRZ modulation format is the most sensitive to the SOA-induced chirp and that of CS-RZ modulation format is almost insensitive to the SOA-induced chirp. Fig.6 shows the eye diagrams of modulation formats in the system with and without SOAs as booster amplifiers with DGD of 50 ps, $\Delta \tau_{\omega}$ of 0.57 ps/GHz, and $|2k|$ of 2.07 °/GHz. As previously mentioned, CS-RZ signals with DGD of 50 ps, $\Delta \tau_{\omega}$ of 0.57 ps/GHz, and $|2k|$ of 2.07 °/GHz are not distorted at all up to 0.4 Å peak-to-peak chirp. However, the NRZ and duobinary NRZ modulation formats have severely distorted in the eye diagrams up to 0.4 Å peak-to-peak chirp.

5. Conclusions

We investigated the influence of the SOA-induced chirp on the PMD tolerance of modulation formats such as NRZ, duobinary NRZ, RZ, and CS-RZ modulation formats. We showed that all the modulation formats have the almost same power penalties, which are induced by the first-order PMD, regardless of the peak-to-peak value of the SOA-induced chirp. In other words, the first-order PMD tolerance is hardly affected by the SOA-induced chirp. On the other hand, the first- and second-order PMD tolerance of all the modulation formats except for the CS-RZ modulation format is affected by the SOA-induced chirp. For a PMD condition of DGD = 50 ps, $\Delta \tau_{\omega}$ = 0.57 ps/GHz, and $|2k|$ = 2.07 °/GHz, the PMD-induced power penalties of the NRZ modulation format increase by 3dB for the peak-to-peak chirp changed from 0 Å to 0.24 Å. For the duobinary NRZ modulation format, the PMD-induced power penalties increase by 3dB when the peak-to-peak chirp increases from 0 Å to 0.31 Å. For the RZ modulation format, the value of the peak-to-peak chirp which makes
PMD-induced power penalty deteriorated by 3dB increases up to 0.6 Å. However, the PMD tolerance of the CS-RZ modulation format is almost not affected by the SOA-induced chirp. In conclusion, in terms of a 3dB additional power penalty, the first- and second-order PMD tolerance of the NRZ modulation format is the most sensitive to the SOA-induced chirp and that of CS-RZ modulation format is insensitive to the SOA-induced chirp.

Fig.6 Eye diagrams for different modulation formats at DGD = 50 ps, \( |\Delta \tau_o| = 0.57 \text{ ps/GHz} \), and \( \text{PMD} = 2.07 \text{ ps/GHz} \); (a) without SOAs as booster amplifiers, (b) peak-to-peak chirp = 0.4 Å

**Acknowledgement**

The work is supported in part by the National Research Laboratory program in Korea.

**References**


