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Synthesis of Linear Arrays with Sidelobe Level Reduction Constraint using Genetic Algorithms

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***Abstract-* The synthesis of uniformly spaced linear array geometries with minimum sidelobe level and beamforming capability using genetic algorithms is presented. The iterative process aims not only at matching the desired pattern to the desired one but minimizing the sidelobe level as well; through optimizing the element excitations. Various examples are included to demonstrate the design effectiveness and flexibility namely for switched smart antenna systems applications.**

***Index Terms*- Linear arrays, pattern synthesis, sidelobe level, genetic algorithms.**

I. INTRODUCTION

In modern wireless communication systems, smart antennas are playing an important role, which may be one of the indispensable techniques in the third and fourth generation systems. Antenna pattern synthesis constitutes an important topic in the smart antenna, in which the speed and robustness of algorithms are the key points [1].

Smart antenna systems, mainly the switched beam type in our application, communicate directionally by forming specific antenna beam patterns and selecting one of them (based on weighted combinations of antenna outputs) with the greatest output power in the remote user's channel [2]. When a smart antenna directs its main lobe with enhanced gain in the direction of the user, it naturally forms side lobes and nulls or areas of medium and minimal gain respectively in directions away from the main lobe. Hence the level of the sidelobes needs to be as mimimum as possible for the sake of interference reduction. Actually, different switched beam and adaptive smart antenna systems control the lobes and the nulls with varying degrees of accuracy and flexibility [2,3].

In general synthesis problems can be treated by classical methods, but each of these is developed in response to a given class of problems. If the radiation pattern has many constraints we have to use numerical optimization techniques. However, we have to take into account the limitations of these methods; especially local minima trap problem, the restrictive requirements of continuity and the existence of derivatives. In the case of large arrays the problem is more critical and the solution space is very large so exhaustive checking of all possible phase-amplitude excitations and-or element positions is very difficult for the methods based on deterministic rules. Hence we must reject calculus-based methods because they are not robust and thus other heuristic algorithms such as Genetic Algorithms (Gas) will be unavoidable. The literature has reported some works about the application of GAs for general electromagnetic problems [2-7] and particularly for the synthesis and design of array antennas [7-13].Genetic algorithms are classified as global optimizers while more familiar, traditional techniques such as conjugate gradient and the quasi-Newtonian methods are classified as local optimizers. The distinction between local and global search of optimization techniques is that the local techniques produce results that are highly dependent on the starting point or initial guess, while the global methods are highly independent of the initial conditions [3-4]. Though they possess the characteristic of being fast in convergence, local techniques, in particular the quasi-Newtonian techniques, have a direct dependence on the existence of at least the first derivative .In addition, they place constraints on the solution space such as differentiability and continuity, conditions that are hard or even impossible to deal with in practice [4]. The global techniques, on the other hand, are largely independent of and place few constraints on the solution space [3]. An advantage of Genetic Algorithms is that they can be used on many problems where traditional techniques cannot be applied easily. This is often the case in problems which are formulated discretely or may contain non-differentiable functions. Another attraction arises in the simplicity of the algorithms and the ease at which they may be programmed. There are many different types of Genetic Algorithms with new techniques being developed constantly. The synthesis of uniform linear array patterns with emphasis on sidelobe level reduction has been considered by some authors in the specialised literature [l1-12-13].

Particularly, the Schelkunoff Polynomial method [12] is very suitable to synthesize uniform linear arrays with a radiation pattern specified by several nulls. In this method the array factor is viewed as a polynomial whose roots are located in the complex w plane; the synthesis is carried out by appropriate placements of the roots in the complex plane. Since there are too many possibilities to achieve the appropriate solution, exhaustive checking of all possible complex excitations is difficult with the optimisation methods that use deterministic rules. Under these conditions GAs acquire importance because they use random search methods, are robust and capable of solving complicated problems [3-4]. Synthesis procedures pertaining to uniformly spaced linear arrays for sidelobe level reduction are quite limited in scope when compared to those for matching the desired radiation pattern only.

In this work, the equally spaced linear arrays are first synthesised using the Schelkunoff method to match an a priori desired pattern. Then, the constraint of sidelobe level reduction is introduced by forcing it in the fitness function.

II.MATHEMATICAL FORMULATION

The array factor for a uniform linear array of N elements with an inter-element spacing d is given by:

$$AF(\theta) = \sum_{n=1}^{N} I_n e^{j(n-1)kd\cos(\theta)} = \sum_{n=1}^{N} I_n e^{j(n-1)\psi} \quad (1)$$

Where:
I_n is the excitation of the element n.
k is the wave number.
d is the spacing between the elements.
Letting:

$$z = x + jy = e^{j\psi} = e^{j(kd\cos(\theta))} \quad (2)$$

Expanding and rearranging we get:

$$AF(\theta) = \sum_{n=1}^{N} a_n z^{n-1} = a_1 + a_2 z + a_3 z^2 + ... + a_N z^{N-1} \quad (3)$$

This is a polynomial of degree *N-1*. From complex variables and algebra, any polynomial of degree *N-1* possesses *N-1* roots and can be expressed as a product of *N-1* terms as follows:

$$AF(\theta) = a_n(z - z_1)(z - z_2)(z - z_3)...(z - z_{N-1}) \quad (4)$$

Where z_1, z_2, z_3,…, z_{N-1} are the roots of the polynomial and may be complex.

The magnitude of the array factor may be expressed as:

$$|AF(\theta)| = |a_n||z - z_1||z - z_2||z - z_3|...|z - z_{N-1}| \quad (5)$$

The complex number z of equation 2 can be written as:

$$z = |z|e^{j\psi} \quad (6)$$

With

$$\psi = kd\cos(\theta) = \frac{2\pi}{\lambda} d\cos(\theta) \quad (7)$$

Some very interesting observations can be drawn from equation 5. When all the roots are not on the unit circle but lie outside or inside of it, only those roots on the circle will contribute to the nulls of the pattern. Hence, it is possible to select the root locations so as to obtain a desired pattern. If the first *M* roots of (5)**,** denoted by w_m' are not on the unit circle then (5) can be written as:

$$AF = \prod_{i=1}^{M}(z - z_i)\prod_{k=M+1}^{N}(z - z_k) \quad (8)$$

By controlling the amplitude and phase of these *M* roots, we can control the shape of the pattern. Other *N-M* roots corresponding to nulls are located over the unit circle in fixed positions.

The synthesis procedure using the Schelkunoff method married with genetic algorithms consists in the following steps. The works is divided into two partas: without sidelobe level reduction constraint and with sidelobe level reduction constraint. In what follows we assume an array with N equally spaced elements. In the Schelkunoff method, an array factor of an array with N elements possesses N-1 roots so that the synthesis procedure for the first part becomes:

1- The radiation pattern is specified using *N-1-M* nulls (where *M* is the number of the roots not lying on the unit circle), and *P* target points. Figure 1 shows an example of how to specify the null locations along with the target points.

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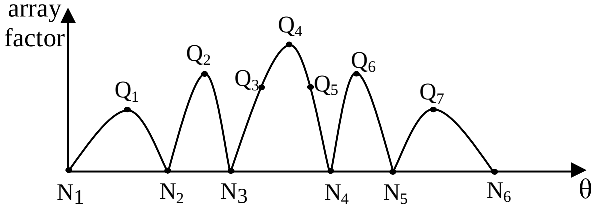

Fig. 1 A sampling scheme of the desired radiation pattern.

2- An individual of dimension *N-1* (the roots of the array factor) is a set of amplitudes and phases of the different roots. These roots include the ones defining the locations of the nulls (called fixed roots) so that the GA varies only those roots not lying on the unit circle.

3- An individual is going to generate a certain radiation pattern that differs from the desired one by an error that is quantified as follows:

$$e_i = \frac{1}{P}\sum_{k=1}^{P}\left|\frac{(Q_{\theta_k} - T_{\theta_k})}{Q_{\theta_k}}\right|^2 \qquad (9)$$

Where Q_{θ_k} is the desired radiation pattern value (target) at angular position θ_k, and T_{θ_k} is the pattern generated by the individual at the same point θ_k.

4- The fitness of an individual is related to the error generated by this individual as follows:

$$F_i = \frac{1}{1+\sqrt{e_i}} \qquad (10)$$

The definition of the fitness this way makes the optimization problem a maximization one rather than a minimization with the error function.

For the second part, the fitness function is modified to contain the sidelobe level term. This is done by introducing a penalty function defined as follows:

$$F_i = \frac{1}{1+\sqrt{e_i}} + F_{sll} \qquad (11)$$

Where F_{sll} is the penalty function introduced to take account for the sidelobe level reduction and is defined as follows

$$F_{sll} = \begin{cases} \dfrac{SLL - SLL_d}{SLL_d} & if \quad SLL > SLL_d \\ 0 & otherwise \end{cases} \qquad (12)$$

The desired sidelobe level has been set to –20 dB which is a typical value for all modern communication systems.

The genetic algorithm was coded in binary, with a population of 80 members (arrays) and an inter-element spacing d=λ/2. The complex roots in the complex plane were coded using 32 bits: 16 for the amplitude and 16 for the phase. The initial population was generated randomly. The selection strategy was the roulette wheel (fitness proportionate selection) and the crossover is the single point crossover. The mutation rate was set to 0.05.

III.RESULTS AND DISCUSSION

The approach presented earlier has been applied to different linear arrays. Particularly, 30-element and 40-element arrays have been chosen as the work in [15] has been concluded to the fact that the Schelkunoff method works better with large arrays. At start up, the two arrays have been synthesized for radiation pattern matching purpose only. Then the sidelobe level constraint has been included. Three direction of the main beam have been chosen for each array: 0°, 30° and –60°. Satisfactory results were achieved. Figures 2.a, 2.b and 2.c show the resulting patterns of the 30-element array along with the samples of the desired pattern having the main beams directed towards 0°, 30° and –60° respectively.

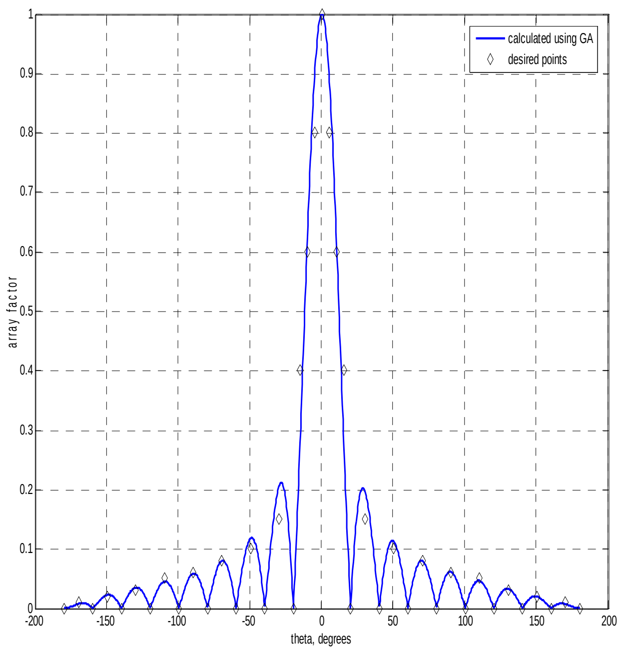

Fig. 2.a. The radiation pattern of the 30- element array having main beam towards 0° along with the desired sample points

Notice that for the three chosen angles, the main beam has been matched at exactly the desired direction. Also, the null placement has been 100% successful. However, the sidelobe level exceeded the beforehand set value and hence needs to be reduced to meet the requirements of modern communication systems.

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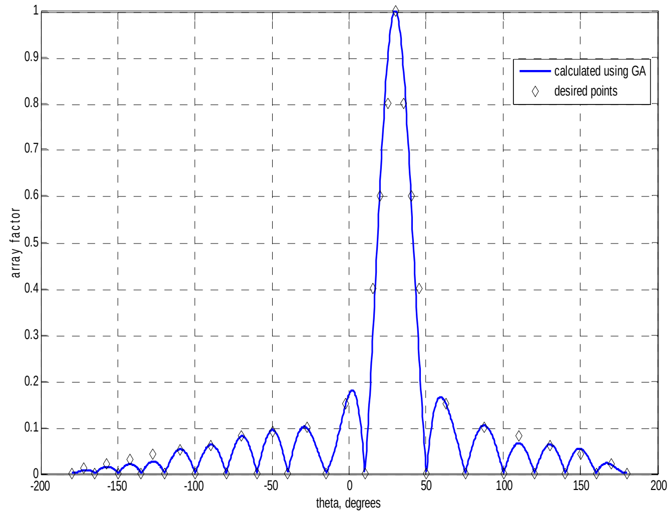

Fig. 2.b. The radiation pattern of the 30- element array having main beam towards 30° along with the desired sample points

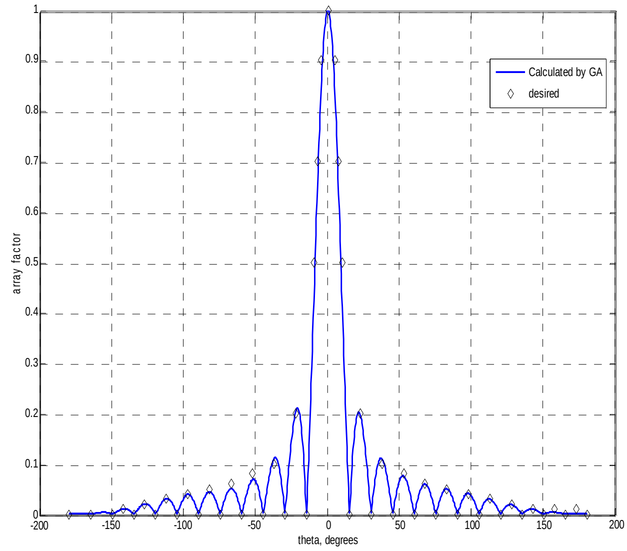

Fig. 3.a. The radiation pattern of the 40- element array having main beam towards 0° along with the desired sample points

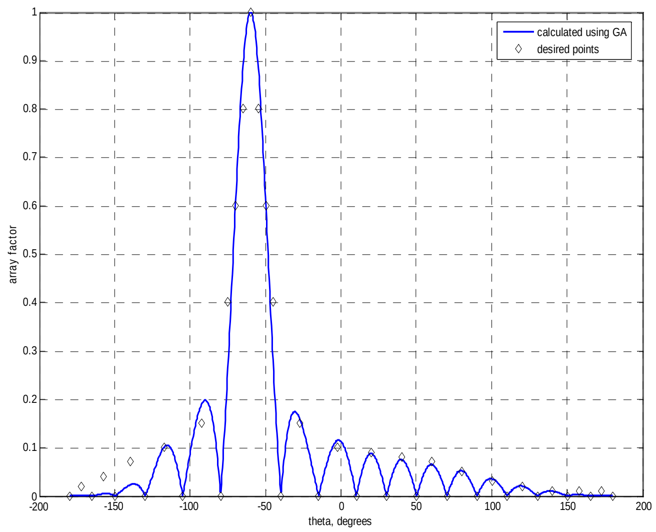

Fig. 2.c. The radiation pattern of the 30- element array having main beam towards –60° along with the desired sample points

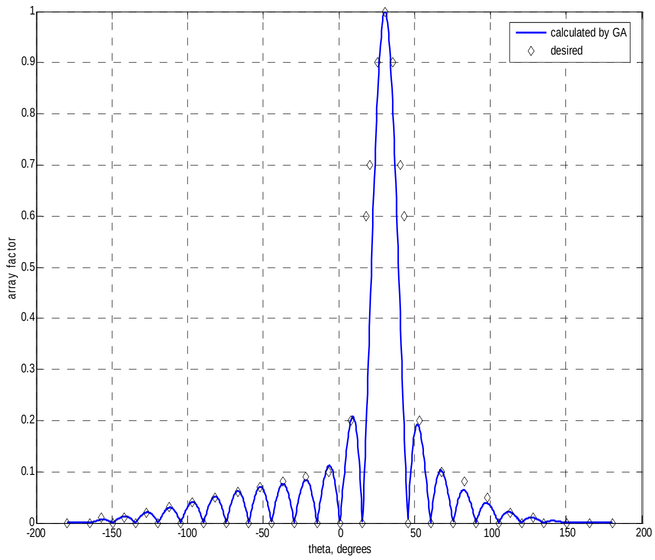

Fig. 3.b. The radiation pattern of the 40- element array having main beam towards 30° along with the desired sample points

For the 40-element array, figures 3.a, 3.b and 3.c show the resulting patterns again for 0°, 30° and –60° as directions of the main beam respectively. It is to be noticed that the directions have been matched perfectly along with a 100% success in placing the nulls. The sidelobe level is less than that of the 30-element array because of imposing the same high value of the directivity of the directivity and knowing the compromise that exits between directivity and sidelobe level, as the number of the elements has increased, the sidelobe level has to decrease. However, the values got still overtake the desired ones and hence addressing the issue of sidelobe level reduction is mandatory.

In conclusion to this part, the exclusion of the sidelobe level from the optimization procedure has outcome fairly satisfactory results and to complete the design and meet the modern reliable system requirements, the sidelobe level has been introduced in the second part using the penalty function defined in equation (12). The same array specifications have been used to have the same basis for comparison and to assess the sidelobe level reduction success or failure. Again, the same three angles have been chosen with the same null and sample point specifications. For the 30-elemen array, the results show a considerable sidelobe level reduction. Figures 4.a, 4.b and

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4.c show the resulting patterns along with the ones obtained in the first part for comparison.

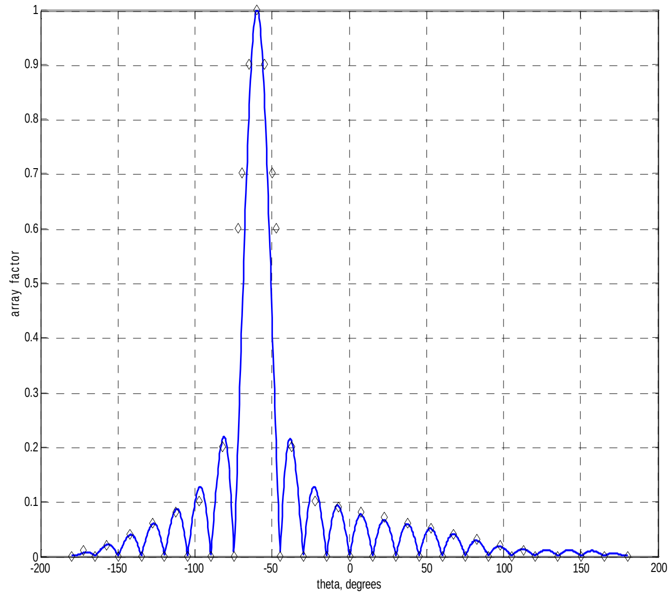

Fig. 3.c. The radiation pattern of the 40- element array having main beam towards –60° along with the desired sample points

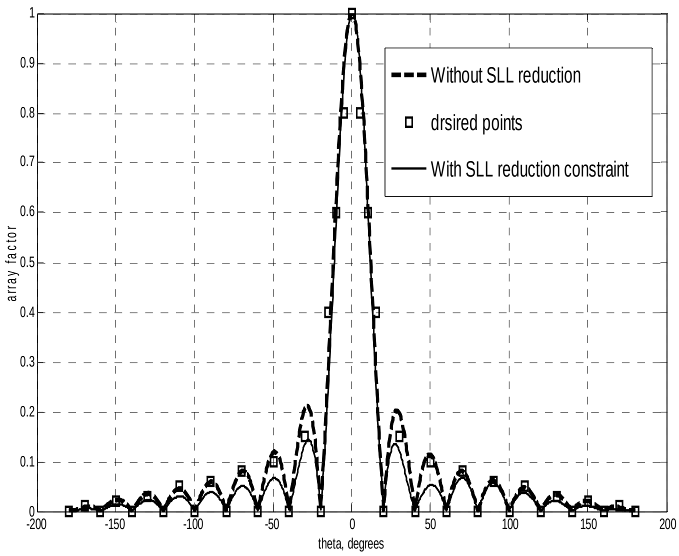

Fig. 4.a. The reduced sidelobe level radiation pattern of the 30-element array (towards 0°) with the previous pattern and the desired sample points

The average sidelobe level reduction relative to the first part was **5.23 dB** with an average sidelobe level value of **21.45 dB** that is suitable for a communication system and is very good especially for the high directivity that has been imposed. For the 40-element array, the resulting patterns are shown in Figures 5.a, 5.b and 5.c. The reduction in sidelobe level was again considerable at an average of **4.12 dB** relative to the results of the first part and the average value reached was **21.54 dB** and again is very adequate for any modern communication system.

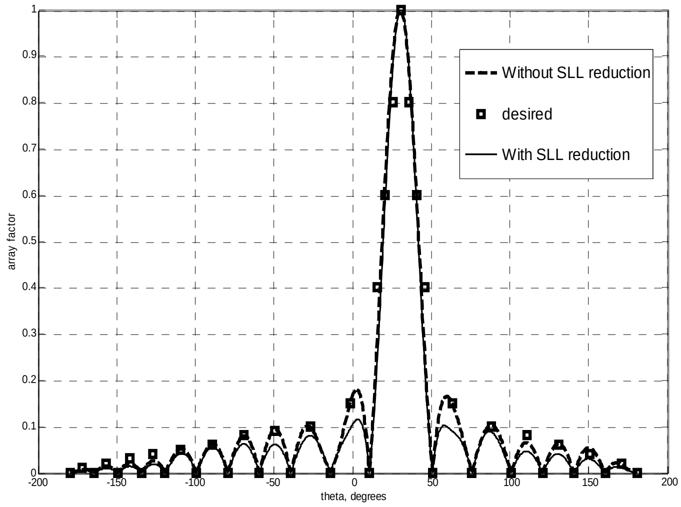

Fig. 4.b. The reduced sidelobe level radiation pattern of the 30- element array (towards 30°) with the previous pattern and the desired sample points

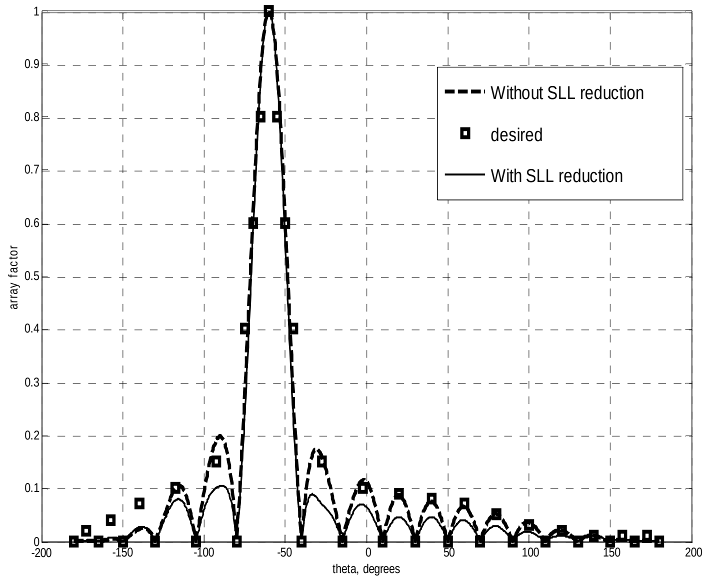

Fig. 4.c. The reduced sidelobe level radiation pattern of the 30-element array (towards –60°) with the previous pattern and the desired sample points

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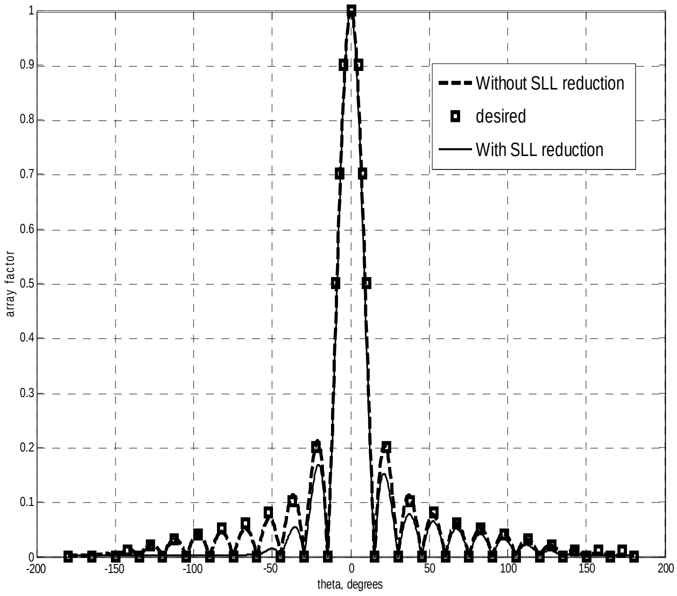

Fig. 5.a. The reduced sidelobe level radiation pattern of the 40-element array (towards 0°) with the previous pattern and the desired sample points

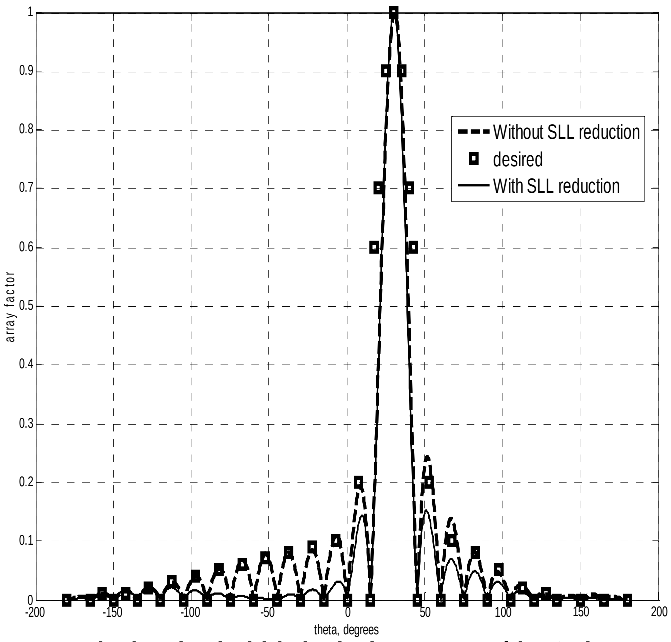

Fig. 5.b. The reduced sidelobe level radiation pattern of the 40- element array (towards 30°) with the previous pattern and the desired sample points

To sum up, the introduction of the sidelobe level reduction into the optimization procedure has led to very interesting and promising results that can be exploited to design reliable communication systems based on the beamforming approach proposed in the present work.

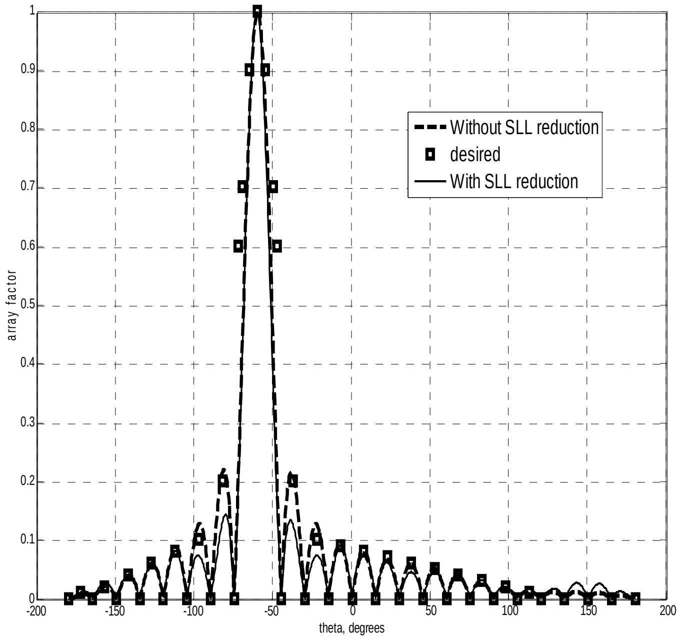

Fig. 5.c. The reduced sidelobe level radiation pattern of the 40- element array (towards –60°) with the previous pattern and the desired sample points

IV.CONCLUSION

The synthesis of equally spaced linear arrays for sidelobe level reduction purpose has been considered in this article. The Schelkunoff method married with Genetic Algorithms has shown up to be useful to synthesize any well designed feasible desired radiation pattern. The method can solve for multi-objective array designs involving many variables namely radiation pattern matching with sidelobe level reduction constraint.

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