

## Physics of Magneto-Optic Waveguide Isolator

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**Abstract-** The paper discusses physical aspects of the waveguide isolator based on the transverse magneto-optical (MO) effect. The isolator operating on TM mode provides a unidirectional transfer of electromagnetic energy from the laser source to the optical fiber. The device consists of an active semiconductor multilayer sandwiched between semiconductor buffer and semiconductor substrate with a MO active ferromagnetic alloy layer deposited on the buffer. The optical gain in the semiconductor multilayer compensates the optical losses due to the reduced reflectivity on the magnetic wall of the waveguide. The characteristics of the device are analyzed with a model planar four-layer waveguide.

**Index Terms-** magneto-optical effects, waveguide isolator, multilayer structures

### I. INTRODUCTION

Unidirectional nature of magneto-optical (MO) Faraday effect forms the basis of non-reciprocal devices, isolators and circulators, widely employed at microwave and optical frequencies. The isolator is indispensable for the protection of the oscillator source, a laser, against spurious reflections of the optical power in the transmission line, which may cause oscillations instabilities. The basic requirement is the highest achievable transmission of the signal in the

forward direction and its maximum attenuation in the backward direction [1].

In his optical activity studies, Fresnel realized that linearly polarized (LP) wave could be decomposed into two oppositely circularly polarized (CP) waves of equal amplitudes. The idea explains the Faraday effect. In a magnetized but otherwise optically isotropic medium parallel to the magnetization vector  $\mathbf{M}$  the CP waves travel with different velocities assigned to them according to their handedness with respect to the orientation of  $\mathbf{M}$ . This forms the basis of the non-reciprocal (uni-directional) action [2]. The picture becomes more complicated in the media with orthorhombic symmetry [3]. This is a consequence of the fact that the proper modes are no more CP but the elliptical ones. A similar effect takes place for the guided modes in planar structures. Here the anisotropy originates from the difference in the reflection coefficients for the LP proper modes classified as TE and TM ones at  $\mathbf{M} = 0$ .

The inspection of the wave equation in isotropic media uniformly magnetized indicates that at the propagation vector perpendicular to the magnetization,  $\mathbf{k} \perp \mathbf{M}$ , specific two pairs of modes result. One pair is linearly polarized with the electric field vector parallel to  $\mathbf{M}$ . The second one is polarized predominantly perpendicular to  $\mathbf{M}$  with a small component parallel to  $\mathbf{k}$ . No MO

effects linear in  $\mathbf{M}$ , which could be considered for the unidirectional operation, exist for the homogeneous waves in the bulk medium. Such an effect, predicted by Wind [5] and experimentally verified by Zeeman [6], called transverse (or equatorial) MO "Kerr" effect (TMOE), occurs at interfaces when a nonuniform (or "leaky") wave is refracted into the magnetic medium. The effect was widely used in the spectroscopic and magnetic domain studies of ferromagnetic metals and ferrimagnetic dielectrics by Krinchik's school [7] and others [8]. Zaets and Ando [9] have exploited the dependence of the phase of the TM reflection coefficient on the transverse magnetization. The selected physical aspects of their isolator design form the subject of the present work. The analytical expressions for the waveguide condition (*i.e.*, the characteristic equation) and the field profiles are provided.

## II. MODEL

The core of the waveguide is formed by a periodic semiconductor structure amplifying transverse magnetic (TM) modes. The non-reciprocal action originates from the MO effect in an optically thick ferromagnetic metallic film magnetically ordered perpendicular to the zigzag wave motion. Under these conditions, the transmission characteristics of the forward TM modes differ from those of the backward ones. An important TMOE advantage with respect to the linear MO effects with polar or longitudinal magnetization consists in its compatibility with the rectangular symmetry of planar and channel waveguides.

A 4x4 matrix formalism provides a tool for the analysis of non reciprocal planar waveguides on anisotropic media with loss and gain consisting of an arbitrary number of layers or of arbitrary tensor permittivity profile (staircase approximation) [10]. Important aspects of the isolator design can however be demonstrated on a four layer model. Such a system is sufficiently simple to be treated starting from the Maxwell equations in planar structures [11]. We consider the system consisting of a semiconductor substrate (0) with the scalar relative permittivity

$\epsilon^{(0)}$ , an amplifying strained multilayer structure modeled by a single semiconductor layer (1) of the thickness  $d_1$  characterized by the relative permittivity tensor

$$\epsilon^{(1)} = \begin{bmatrix} \epsilon_{xx}^{(1)} & 0 & 0 \\ 0 & \epsilon_{yy}^{(1)} & 0 \\ 0 & 0 & \epsilon_{zz}^{(n)} \end{bmatrix} \quad (1)$$

where  $\epsilon_{yy}^{(1)} = \epsilon_{zz}^{(1)} \equiv \epsilon_0^{(1)}$ , buffer interlayer (2) of thickness  $d_2$ , with the scalar relative permittivity  $\epsilon^{(2)} = \epsilon^{(0)}$  in contact with a magnetic layer (3) characterized by the relative permittivity tensor

$$\epsilon^{(3)} = \begin{bmatrix} \epsilon_3^{(3)} & 0 & 0 \\ 0 & \epsilon_0^{(3)} & i\epsilon_1^{(3)} \\ 0 & -i\epsilon_1^{(3)} & \epsilon_0^{(3)} \end{bmatrix} \quad (2)$$

Both the substrate and magnetic layer are optically thick. A Cartesian coordinate system with the  $x$ -axis parallel to the transverse magnetization, the propagation parallel to the  $y$ -axis and the interface planes normal to the  $z$ -axis is chosen. The normalized longitudinal propagation constant  $N_y$ , for TM modes follows from the characteristic equation

$$1 + r_{pp}^{(01)} \left[ \frac{r_{pp}^{(12)} + r_{pp}^{(23)} \exp(-2i\kappa^{(2)}d_2)}{1 + r_{pp}^{(12)} r_{pp}^{(23)} \exp(-2i\kappa^{(2)}d_2)} \right] \exp(-2i\kappa^{(1)}d_1) = 0 \quad (3)$$

where

$$r_{pp}^{(01)} = \frac{\epsilon_0^{(0)} N_z^{(1)} - \epsilon_0^{(1)} N_z^{(0)}}{\epsilon_0^{(0)} N_z^{(1)} + \epsilon_0^{(1)} N_z^{(0)}}, \quad r_{pp}^{(12)} = \frac{\epsilon_0^{(1)} N_z^{(2)} - \epsilon_0^{(2)} N_z^{(1)}}{\epsilon_0^{(1)} N_z^{(2)} + \epsilon_0^{(2)} N_z^{(1)}} \quad (4)$$

$$r_{pp}^{(23)} = \frac{\epsilon_0^{(2)} N_z^{(3)} - \epsilon_0^{(3)} N_z^{(2)} \left( 1 - \frac{i\epsilon_1^{(3)} N_y}{\epsilon_0^{(3)} N_z^{(3)}} \right)}{\epsilon_0^{(2)} N_z^{(3)} + \epsilon_0^{(3)} N_z^{(2)} \left( 1 - \frac{i\epsilon_1^{(3)} N_y}{\epsilon_0^{(3)} N_z^{(3)}} \right)} \quad (5)$$

The reflection coefficient can be written as  $r_{pp}^{(23)} = \exp(i\theta_{pp}^{(23)})$ . Figure 1 shows its dependence

on the angle of incidence. The magneto-optic part of  $\theta_{pp}^{(23)}$  is shown in Fig. 2.

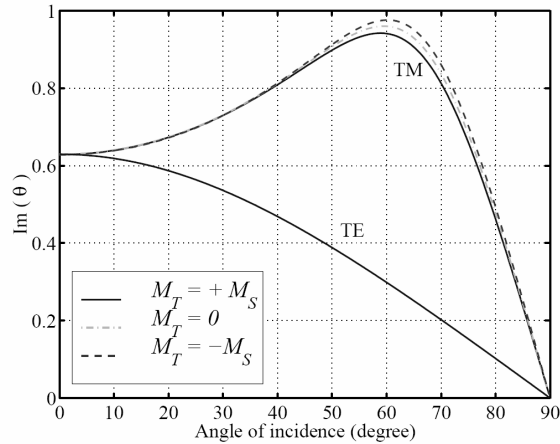


Fig. 1 Imaginary part of the TM complex phase shift  $\theta_{pp}^{(23)}$  computed for an interface between InP

with the relative permittivity  $\epsilon^{(2)} = 3.4^2$  and CoFe alloy with  $\epsilon_0^{(3)} = (4.35-i4.75)^2$  and  $\epsilon_1^{(3)} = 0.3030+i0.0906$ . The perturbations induced by the transverse magnetization with positive and negative polarity are included. The TE case is shown for comparison.

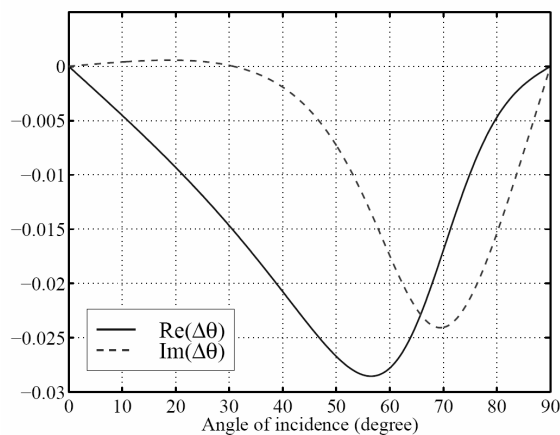


Fig. 2 Magneto-optic phase complex shift  $\Delta\theta_{pp}^{(23)}$  computed for an interface between InP with the relative permittivity  $\epsilon(2) = 3.42$  and CoFe alloy with  $\epsilon_0(3) = (4.35-i4.75)^2$  and  $\epsilon_1(3) = 0.3030+i0.0906$ .

The transverse propagation constants in the layers are defined as  $\kappa^{(j)} = \frac{\omega}{c} (\epsilon_0^{(j)} - N_y^2)^{1/2} = \frac{\omega}{c} N_z^{(j)}$

for  $j = 0, 1 \dots 4$ , where  $\omega$  and  $c$  denote the angular frequency and the vacuum phase velocity of the monochromatic wave. We observe that the solution of Eq. (3) depends on the product  $\epsilon_1^{(3)} N_y$ , which changes sign when either  $\mathbf{M}$  or  $N_y$ , changes its sign. We then have either  $\epsilon_1^{(3)}(M) \rightarrow -\epsilon_1^{(3)}(-M)$  upon the  $\mathbf{M}$  reversal or  $N_y \rightarrow -N_y$  upon the reversal of the propagation sense. In semiconductor InP structure waveguides, the permittivity of the active layer and that of substrate (and buffer interlayer) assume typically the values  $\epsilon^{(1)} \approx 3.4^2$  and  $\epsilon^{(0)} = \epsilon^{(2)} \approx 3.2^2$ , respectively. Then, the critical angle for total reflection occurs at nearly 70 deg. This puts the restriction on the longitudinal propagation constant  $\beta = (\omega/c) N_y > (\omega/c) \epsilon^{(0)1/2}$ . Note, that in this region, TMOKE is a decreasing function of the angle of incidence as seen in Fig. 1 and 2.

We therefore choose  $\epsilon_0^{(1)} > \epsilon_0^{(2)}$ ,  $\epsilon_0^{(1)} > \epsilon_0^{(2)}$  and assume the transverse propagation constant in the layer region (1) to be an almost real quantity  $\kappa^{(1)} \approx (\epsilon_0^{(1)} - N_y^2)^{1/2}$ ,  $\omega/c > 0$ , while the transverse attenuation constant in the region (0) satisfies  $\gamma^{(0)} = -i\kappa^{(0)} = (N_y^2 - \epsilon_0^{(0)})^{1/2}$ ,  $\omega/c > 0$ , and  $\gamma^{(0)} = \gamma^{(2)}$ . Equation (3) characterizes a symmetric waveguide and no gain in (1) is necessary to allow guided mode propagation. The reflection coefficients  $r_{pp}^{(10)} = -r_{pp}^{(01)} = \exp(i\theta^{(10)})$  and  $r_{pp}^{(12)} = \exp(i\theta^{(12)})$  at the boundary of the active region (1) are approximately of unit magnitude and the corresponding phase angles  $\theta^{(10)}$  and  $\theta^{(12)}$ , are practically real. In absorbing media, the phase angles are complex. We define

$$\exp(i\theta^{(123)}) = \frac{r_{pp}^{(12)} + r_{pp}^{(23)} \exp(-2i\kappa^{(2)}d_2)}{1 + r_{pp}^{(12)} r_{pp}^{(23)} \exp(-2i\kappa^{(2)}d_2)} \quad (6)$$

Equation (3) requires for the real part

$$\Re(\theta^{(10)}) + \Re(\theta^{(123)}) - 2d_1 \frac{\omega}{c} \Re[(\epsilon_0^{(1)} - N_y^2)^{1/2}] = 2\pi\nu \quad (7a)$$

and for the imaginary part

$$\Im(\theta^{(10)}) + \Im(\theta^{(123)}) - 2d_1 \frac{\omega}{c} \Im[(\epsilon_0^{(1)} - N_y^2)^{1/2}] = 0 \quad (7b)$$

where  $\nu = 0, 1 \dots$  numbers the modes. The solution to Eqs. (7) provide  $N_y$  and  $\Im[(\epsilon_0^{(1)} - N_y^2)^{1/2}]$ . The latter represents the gain required for non-attenuated mode propagation. We are concerned with the fundamental mode  $\nu = 0$ . With  $N_y$ , known we can determine the field profiles. Up to a common factor, the magnetic fields  $H_x$  become

$$H_x^{(0)} = (1 + r_{pp}^{(01)}) e^{\gamma^{(0)}z}, \quad z < 0 \quad (8a)$$

$$H_x^{(1)} = e^{i\kappa^{(1)}z} (1 + r_{pp}^{(01)} e^{-2i\kappa^{(1)}z}), \quad 0 < z < d_1 \quad (8b)$$

$$H_x^{(2)} = \frac{e^{i\kappa^{(1)}z}}{1 + r_{pp}^{(12)}} [(r_{pp}^{(12)} + r_{pp}^{(01)} e^{-2i\kappa^{(1)}d_1}) e^{-i\kappa^{(2)}(z-d_1)} + (1 + r_{pp}^{(01)} r_{pp}^{(12)} e^{-2i\kappa^{(1)}d_1}) e^{i\kappa^{(2)}(z-d_1)}], \quad d_1 < z < d_1 + d_2 \quad (8c)$$

$$H_x^{(3)} = \frac{e^{i\kappa^{(1)}z}}{1 + r_{pp}^{(12)}} [(r_{pp}^{(12)} + r_{pp}^{(01)} e^{-2i\kappa^{(1)}d_1}) e^{-i\kappa^{(2)}d_2} + (1 + r_{pp}^{(01)} r_{pp}^{(12)} e^{-2i\kappa^{(1)}d_1}) e^{i\kappa^{(2)}d_2}] e^{-i\kappa^{(3)}(z-d_1-d_2)}, \quad d_1 + d_2 < z \quad (8d)$$

The corresponding electric fields follow from the Maxwell equations written here to first order in  $\epsilon_1^{(3)}$

$$E_y^{(3)} = \frac{-i}{\omega \epsilon_{vac} \epsilon_0^{(3)}} \left( \frac{dH_x^{(3)}}{dz} + \beta \frac{\epsilon_1^{(3)}}{\epsilon_0^{(3)}} H_x^{(3)} \right) \quad (9a)$$

$$E_z^{(3)} = \frac{1}{\epsilon_0^{(3)} \omega \epsilon_{vac}} \left( \beta H_x^{(3)} + \frac{\epsilon_1^{(3)}}{\epsilon_0^{(3)}} \frac{dH_x^{(3)}}{dz} \right) \quad (9b)$$

where  $\epsilon_{vac}$  denotes the vacuum permittivity.

### III. DISCUSSION

The optimum operation of the isolator includes a high reflected-mode rejection. Figure 3 shows the separation of the forward and backward modes computed for the four-layer model. The efficient separation requires a significant complex MO phase shift component (dependent on the propagation sense). It is desirable that the isolator have low insertion loss and operates at the fundamental TM mode with a cosine profile of the transverse field *i.e.*, with the Poynting vector maximum located at the central plane of the periodic semiconductor structure. This profile is close to that of the fundamental mode in monomode optical fibers. A high effective reflectivity at the active layer interfaces would allow reducing the gain necessary for the non-attenuated mode propagation. As observed in Figures 1 and 2, a high reflectivity and strong MO coupling are in conflict and a suitable compromise should be found. The permittivity ratio of semiconductor active layer and substrate sets the total reflection critical angle of incidence to 70 deg, *i.e.*, to the region where the TM wave reflectivity is low. The real part of the MO phase shift at InP-CoFe alloy interface has its maximum below 60 deg. (Fig. 2). There is some space for an improved performance via  $d_2$  tuning. The performance of the device could also be improved by the reduction of the critical angle using lower  $\epsilon^{(2)}$  layer. The MO phase shift can be doubled in the symmetric configuration with two magnetic layers with opposite transverse magnetization sandwiching the semiconductor waveguide structure.

The structure employed in practice will be built on a rectangular rather than planar guide. There, the TE and TM mode classification becomes approximate only. A reduced symmetry of the strained semiconductor layers makes the permittivity pertinent to the TM and TE modes

different, which may improve the rejection of the nearly degenerate TE mode. The presence of the spontaneous magnetization transforms both the permittivity and magnetic permeability to a tensor. Therefore, there is an analogous TMOE on TE waves originating from the magnetic dipole electron transitions at microwave and far infrared frequencies. The effect is weak but can be in principle enhanced in the nanostructured materials.

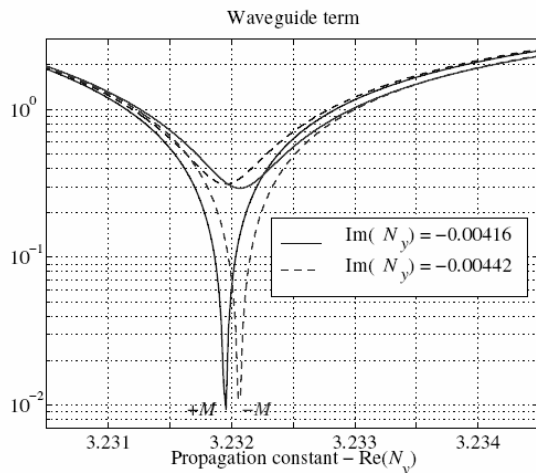


Fig. 3 The right hand of Eq. (3) plotted as a function of the real and imaginary part of the reduced longitudinal propagation  $N_y$  constant for the four-layer system of reference [1]. The situation with reversed magnetization indicates the separation of the forward and reversed modes in the isolator.

In conclusion, we have provided information for the optimum design of the waveguide isolator based on transverse magneto-optic effect in an analytical form. The explicit waveguide condition for a four layer system with the corresponding field profile, and the expressions for the TMOE reflection at a single interface and in the film surface system help to choose candidate materials and the thickness of the active guiding and coupling layers leading to a design with an optimized performance.

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