Analytically Characterization and Optimum Performance of Nonlinearly Distorted Coherent Optical OFDM Signals

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Abstract - Nonlinear phase noise can severely degrade the performance of multicarrier optical signals by introducing both inband and out-of-band distortion. In this paper we study analytically the impact of nonlinear phase distortion on CO-OFDM (Coherent Optical Orthogonal Frequency Division Multiplexing) schemes with large number of subcarriers. By using the Bussgang theorem, we derive the power spectral densities of the useful and the distortion term of the nonlinearly distorted signal. The traditional approach in conventional CO-OFDM implementations is to treat nonlinear distortion as an undesirable noise term that leads to performance degradation. However, it can be shown that this distortion has information on the transmitted signals that can be used to improve the performance. Surprisingly, we show that the distortion inherent to the nonlinear phase noise can lead to performance improvements relatively to the ideal, linear CO-OFDM transmissions. We present a closed-form expression for the average asymptotic gain associated to the optimum detection of nonlinearly distorted CO-OFDM signals and we derive the approximated bit-error-rate using the distribution of the asymptotic gain.

Index terms: optical communications, optimum detection, multicarrier signals, nonlinear phase noise.

I. INTRODUCTION

CO-OFDM (Coherent Optical Orthogonal Frequency Division Multiplexing) schemes [1] are widely used in long-haul wired communications, mainly due to their facility to achieve high data rates without substantial equalization efforts. However, as other OFDM schemes, the transmitted signals have high envelope fluctuations and a high PAPR (Peak-to-Average Power Ratio), making them prone to nonlinear distortion effects [2-3]. In addition, the CO-OFDM schemes are particularly sensitive to NLPN (NonLinear Phase Noise), that results from the interaction between the ASE (Amplified Spontaneous Emission) and the transmitted signal through a Kerr nonlinearity [4-5].

Several techniques involving pre-distortion and/or post-compensation of the nonlinear distortion have been proposed to alleviate the NLPN problem [6]. Alternatively, we can simply accept the existence of nonlinear effects in the transmission. When the number of subcarriers is high the CO-OFDM signals have a Gaussian-like nature [7-9], which means that the nonlinearly distorted signals can be decomposed as the sum of uncorrelated useful and self-interference components [10]. Conventional receiver implementations treat the nonlinear self-interference component as an additional noise term that leads to performance degradation [11]. However, this component has information on the transmitted signal that can be used to improve the performance [12]. Recent results indicate that the optimum performance in the presence of strong clipping characteristics, and, consequently, strong nonlinear distortion effects, can even be better than the performance with ideal, linear transmitters [13-14]. However, the nonlinearity that models the NLPN is substantially different from the nonlinear clipping characteristics considered in those works. Therefore, one might wonder what is the achievable performance of CO-OFDM in the presence of strong nonlinear phase distortion effects.

In this paper we consider CO-OFDM transmission...
with strong NLPN distortion effects. We characterize the nonlinear CO-OFDM signals in the frequency domain by presenting results of their PSD (Power Spectral Density) and we study the performance of an optimum receiver by presenting the distribution of the asymptotic gain and a closed-form expression for the average asymptotic gain. Moreover, results of the approximate BER (Bit Error Rate) are included for both dispersive and non-dispersive channels. The results confirm that the nonlinear phase distortion can lead to performance improvements relatively to the ideal linear case, contrary to what one could expect. This optimum performance can be achieved with relatively simple sub-optimum receivers.

This paper is organized as follows: in Sec. 2, the baseband nonlinear CO-OFDM chain that is considered in this work is presented. An analytical characterization of nonlinearly distorted CO-OFDM signals is made in Sec. 3. In Sec. 4 we study the potential performance of optimum receivers that deal with nonlinearly distorted CO-OFDM signals and we derive a closed-form expression for the average asymptotic gain and present approximate BER results. Sec. 5 concludes this paper.

Throughout the paper we adopt the following conventions: capital letters are associated to the frequency domain and small letters are associated to the time domain. \( \| X \| \) denotes the Euclidean norm of the vector \( X \) and \( (\cdot)^T \) denote the transpose operator. The PDF (Probability Density Function) of the random variable \( x \), \( p_s(x) \), is simply denoted by \( p(x) \) when there is no risk of ambiguity. \( \mathbb{E}[\cdot] \) denotes the average value.

II. NONLINEARLY DISTORTED CO-OFDM SIGNALS

In this section we characterize the signals along the nonlinear CO-OFDM chain represented in Fig. 1. The input data signal is represented by the vector \( S = [S_0, S_1, ..., S_{NM-1}]^T \in \mathbb{C}^{NM} \), which is a frequency-domain vector with \( N \) complex data symbols selected from a given constellation according to the transmitted data bits and a given mapping rule, plus \( (M-1)N \) idle subcarriers at the edges of the useful band to perform the oversampling operation (\( M \) represents the oversampling factor). A time-domain version of \( S \) is given by

\[
S = F^{-1}S = [s_0, s_1, ..., s_{NM-1}]^T \in \mathbb{C}^{NM}, \quad \text{with } F
\]

denoting the \( NM \)-point DFT (Discrete Fourier Transform) matrix, i.e,

\[
F_{n,m} = \frac{1}{\sqrt{NM}} \exp \left( -\frac{j2\pi nm}{NM} \right). \tag{1}
\]

When the CO-OFDM block has a large number of sub-carriers, the correspondent time-domain signal is accurately modeled by a complex Gaussian random process. In these conditions, the real and imaginary parts of time-domain sample \( s_n \) can be modeled by a Gaussian random variable with zero mean and a given variance \( \sigma^2 \), \( s \sim \mathcal{N}(\mu, \sigma^2) \). The absolute value of \( s \) is represented by \( R = [R_0, R_1, ..., R_{NM-1}]^T \in \mathbb{R}^{NM} \), whose the elements can be modeled with \( R \), a Rayleigh random variable with distribution

\[
p(R) = \frac{R}{\sigma^2} \exp \left( -\frac{R^2}{2\sigma^2} \right) \mu(R), \tag{2}
\]
where $u(R)$ denotes the Heaviside function and $\sigma^2$ is the variance of $s$. The nonlinearly distorted CO-OFDM signal is represented by $y = [y_0 \ y_1 \ ... \ y_{NM-1}]^T \in \mathbb{C}^{NM}$, whose the $n^{th}$ element $y_n$ is given by

$$y_n = f(R_n) \exp(j \theta_n) = A(R_n) \exp(j(\Theta(R_n) + \theta_n)), $$

where $A(\cdot)$ and $\Theta(\cdot)$ denote the AM-AM and AM-PM conversion characteristics of the nonlinearity and $\theta_n$ represents the original phase of the $n^{th}$ sample. Thanks to the Gaussian nature of the CO-OFDM signal and using the so-called Bussgang Theorem [10], we can separate the nonlinearly distorted signal into two components: one proportional to the input signal and another one that represents the nonlinear distortion effects, i.e.,

$$y_n = \alpha s_n + d_n, $$

where $d_n$ is the $n^{th}$ sample of the self-interference component introduced by the nonlinearity and $\alpha$ is a scaling factor given by

$$\alpha = \frac{\mathbb{E}[y_n s_n^*]}{\mathbb{E}[s_n^2]} = \frac{\mathbb{E}[R_n f^*(R_n)]}{\mathbb{E}[R_n^2]}, $$

By multiplying the DFT matrix by $y$ we obtain the frequency-domain version of the nonlinearly distorted CO-OFDM signal, that is

$$Y = F y = \alpha S + D, $$

where $D = F d = [D_0 \ D_1 \ ... \ D_{NM-1}]^T \in \mathbb{C}^{NM}$ is the frequency-domain version of the distortion term introduced by the nonlinearity. The NLPN considered in this work can be modeled as a polar memoryless nonlinearity [15] and described by the following AM-AM characteristic function

$$A(R) = R, $$

and the following AM-PM characteristic function

$$\Theta(R) = 2\pi k_\theta R. $$

The parameter $k_\theta$ is related to the magnitude of NLPN distortion effects. When there is a linear transmission we have $k_\theta = 0$ and, consequently, $\Theta(R) = 0$. As the nonlinear distortion effects increase, the absolute value of $\alpha$ and, consequently, the power of the useful component (proportional to $|\alpha|^2$) decreases as it can be seen in Fig. 2.

Fig. 2. Evolution of $|\alpha|^2$ with $k_\theta$.

As we are working with Gaussian random signals whose the variance depends on the number of subcarriers $N$ and on the oversampling factor $M$, we consider to use a normalized NLPN parameter given by $k_\theta/\sigma$. In these conditions, (3) yields

$$y_n = R_n \exp(j(2\pi k_\theta \sigma_\theta/\sigma + \theta_n)). $$

III. ANALITCALLY CHARACTERIZATION OF NONLINEARLY DISTORTED CO-OFDM SIGNALS

To understand the impact of the NLPN distortion on the CO-OFDM signal is important to see the
how the signal is affected both in time and in frequency-domain. In this section, by taking advantage of the Gaussian nature of the CO-OFDM signals, we present closed-form expressions for their autocorrelation and for the PSD in the presence of NLPN distortion. Let us start by defining the auto-correlation of the input signal is given by

$$ R_{x,n-n'} = E[s_n s^*_{n'}] . \quad (10) $$

Using the expected value properties, we also can write

$$ R_{x,n-n'} = \frac{1}{(NM)^2} \sum_{k=0}^{NM-1} \sum_{k'=0}^{NM-1} E[S_k S_{k'}^*] \times \exp \left(-j2\pi \frac{kn-k'n'}{NM} \right). \quad (11) $$

Considering $k' = k$, we have

$$ R_{x,n-n'} = \frac{1}{(NM)^2} \sum_{k=0}^{NM-1} G_{x,k} \times \exp \left(-j2\pi \frac{k(n-n')}{NM} \right). \quad (12) $$

When $n = n'$ we obtain the average power of the baseband OFDM signal, given by

$$ R_{x,0} = E[|s_n|^2] = 2\sigma_x^2 = 2\sigma_F \frac{N}{NM} $$

$$ \sigma_F = E[|S_k|^2/2] = 1 \quad \text{assuming normalized QPSK (Quadrature Phase Shift Keying) constellations with } S_k = \pm 1 \pm j. \quad \text{From (12), it is clear that the auto-correlation and the PSD form a Fourier pair } \left( R_{x,n} = \frac{1}{NM} \text{IDFT}(G_{x,k}) \right). \quad \text{In [7], it is shown that the auto-correlation of an OFDM signal distorted by a polar nonlinearity can be written as a function of the input autocorrelation, } R_{x,n}, \text{ resulting,} $$

$$ R_{y,n-n'} = 2 \sum_{y=0}^{\gamma+1} P_{2\gamma+1} \frac{R_{y,n-n'}^{2\gamma+1} R_{x,n-n'}^{2\gamma}}{R_{x,0}^{2\gamma+1}}, \quad (13) $$

where $P_{2\gamma+1}$ denotes the power associated to the intermodulation product of order $2\gamma+1$, defined as

$$ P_{2\gamma+1} = \frac{1}{4\sigma_F^6 (\gamma+1)} \times \left| \int_0^{\infty} R^2 f(R) \exp \left(-\frac{R^2}{2\sigma^2}\right) L_{\gamma}^{(1)} \left( \frac{R^2}{2\sigma^2} \right) dR \right|^2, \quad (14) $$

where $L_{\gamma}^{(1)}$ is the Laguerre Polynomial of $\gamma$ degree. The PSD of a nonlinearly distorted CO-OFDM signal is simply the DFT of its autocorrelation, i.e., $G_{y,k} = \text{DFT}(R_{y,n})$. In Fig. 3 is shown the PSD of a nonlinearly distorted CO-OFDM signal considering $N = 128$, an oversampling factor of $M = 4$ and $k_0 = 0.1$. The PSDs were obtained both theoretically (using (14)) and by simulation.

Fig. 3. PSD of the nonlinearly distorted CO-OFDM signal considering $k_0 = 0.1$ and $n_\gamma = 1$ (top figure) and $n_\gamma = 10$ (bottom figure) intermodulation products.

From the figure, we note that the higher the number of intermodulation products that are considered, the accurate are the results. Although $n_\gamma = 1$ is not enough for obtaining a good
estimate of the PSD, when \( n_r = 10 \) the PSD obtained by simulation shows complete agreement with the one obtained theoretically. In addition, the higher the order of the intermodulation product, the smaller is its contribution to the output PSD. As in (4), it is also possible to separate the output PSD into two PSDs, one related to the useful part of the signal and another associated to its distortion term, i.e.,

\[
G_{y,k} = |\alpha|^2 G_{s,k} + G_{d,k}.
\]

(15)

The useful power is given by \( P_1 \), i.e., by the first intermodulation product (that can obtained with (15) and \( \gamma = 0 \)). The distortion power is given by \( \sum_{\gamma=1}^{\infty} P_{2\gamma+1} \), hence, the PSD of the distortion component, \( G_{d,k} \), can be obtained by applying the DFT to the autocorrelation of the distortion component, that is given by

\[
R_{d,n-n'} = 2\sum_{\gamma=1}^{\infty} P_{2\gamma+1} \frac{R_{s,n-n'} \Gamma_{\gamma}^2}{R_{s,0} \Gamma_{\gamma+1}^2}.
\]

(16)

Fig. 4 shows the PSD of the distortion component, considering a CO-OFDM signal with \( N = 128 \) and \( M = 4 \). Moreover, we consider two values of \( k_\theta \).

![Fig. 4. PSD of the distortion term considering two values of \( k_\theta \) and \( n_r = 10 \) intermodulation products.](image)

With these values of \( k_\theta \) we note that \( n_r = 10 \) intermodulation products are enough to obtain an accurate PSD. From the figure, it can also be noted that for low values of \( k_\theta \) the distortion levels at the subcarrier level are also lower. Conventional receivers assume that the nonlinear distortion is an additional noise term that is added to thermal noise. Therefore it can lead to substantial performance degradation.

The conventional CO-OFDM receivers treat the nonlinear distortion as a noise term that degrades the performance of the system. This effect can be seen in Fig. 5, where it can be noted that in the presence of NLPN distortion, the original QPSK constellation points will be shifted and rotated, which is traduced in BER degradations.

![Fig. 5. Received QPSK symbols with \( k_\theta = 0.05 \) and without \( k_\theta = 0 \) NLPN distortion.](image)

An estimate of this degradation can be obtained through the SIR levels at a given subcarrier that is defined as

\[
SIR = \frac{|\alpha|^2 E[|S_k|^2]}{E[|D_k|^2]}.
\]

(17)
Fig. 6. Evolution of the SIR for $k_\theta = 0.15$, $k_\theta = 0.2$ and $n_f = 10$ intermodulation products.

IV. OPTIMUM PERFORMANCE OF NONLINEARLY DISTORTED CO-OFDM SIGNALS

The work presented in [13-14] unveils that the existence of nonlinear distortion effects combined with the use of optimum receivers can provide substantial gains relatively to the conventional, linear OFDM schemes. In order to understand this, let us consider two undistorted data sequences

$$\begin{align*}
S^{(1)} &= [S_0^{(1)} S_1^{(1)} \ldots S_{NM-1}^{(1)}]^T \in \mathbb{C}^{NM}\quad \text{and} \\
S^{(2)} &= [S_0^{(2)} S_1^{(2)} \ldots S_{NM-1}^{(2)}]^T \in \mathbb{C}^{NM}
\end{align*}$$

that differ in only one bit. When we don’t have nonlinear distortion and the transmission is linear, the Euclidean distance between these two sequences is given by $D^2 = 4E_b$. However, as seen in Fig. 3, the nonlinear distortion effects cause a spectral broadening and the energy is spread across the signal bandwidth. In these conditions, the difference between the nonlinearly distorted sequences has energy not only in the subcarrier where the bit was modified. In fact, the Euclidean distance between two nonlinearly distorted signals $Y^{(1)}$ and $Y^{(2)}$ (the distorted version of $S^{(1)}$ and $S^{(2)}$, respectively) is

$$D^2 = \sum_{k=0}^{NM-1} |Y_k^{(1)} - Y_k^{(2)}|^2 = \sum_{k=0}^{NM-1} |D_k^{(1)} + D_k^{(2)}|^2,$$

a value that is typically greater than $4E_b$, which leads to an asymptotic gain relatively to a linear CO-OFDM transmission that can be expressed as

$$G = \frac{D^2}{4E_b}.$$  \hspace{1cm} (18)

Fig. 7 shows the distribution of $G$ considering different values of $k_\theta$ and CO-OFDM sequences that have $N = 64$, $M = 4$ and differ in only one bit.

From the figure, is easily observed that the average value of the gain increases when stronger nonlinear distortion effects are considered. Moreover, its variance decreases as $N$ increases.

In the following we present an analytical method for obtaining the average Euclidean distance between two nonlinearly distorted CO-OFDM signals that differ in a single bit. Let us consider the frequency-domain data block, $X^{(2)}$, that differs from $X^{(1)}$ in only one bit at the $k$th subcarrier and the corresponding time-domain blocks $x^{(2)} = F^{-1}X^{(2)}$ and $x^{(1)} = F^{-1}X^{(1)}$, respectively. We can say that
where the matrix $E = [E_0, E_1, \ldots, E_{N-1}]^T \in \mathbb{C}^{N \times M}$ can be regarded as an error term. For QPSK constellations with symbols $X_k = \pm \beta \pm j\beta$ and considering $N$ in-band subcarriers, we have

$$E_k = \begin{cases} \pm 2\beta \text{ or } \pm 2j\beta, & k = k_0 \\ 0, & k \neq k_0 \end{cases}$$

(20)

The time-domain version of this error term is $e = [e_0, e_1, \ldots, e_{N-1}]^T \in \mathbb{C}^{N \times M} = F^{-1}E$. Since $E_k = 0$ for all subcarriers except at $k = k_0$, we have the $n$th element of $e$ defined as

$$e_n = \frac{2\beta}{\sqrt{NM}} \exp\left(\frac{j2\pi nk_0}{NM} + j\arg(E_{0})\right) = \Delta \exp(j\phi_n),$$

(21)

with $\Delta = 2\beta/\sqrt{NM}$. From (10), we can assume that $\phi_n$ also has uniform distribution in $[0, 2\pi]$. Moreover, $R_n$, $\theta_n$ and $\phi_n$ are assumed to be uncorrelated. From (8), we have $x^{(2)} = x^{(1)} + e$. Since typically the term $e$ is much lower than $x^{(1)}$ and, consequently, much lower than $x^{(2)}$, we can say that

$$R' \exp(j\theta') = R \exp(j\theta) + \Delta \exp(j\phi),$$

(22)

where $R'$ and $\theta'$ are the random variables associated to the absolute value and phase of the samples of $x^{(2)}$. Due to the circular nature of $x^{(1)}$ and $e$, we can assume without loss of generality that $\theta = 0$, leading to

$$R' \exp(j\theta') = (R + \Delta \cos(\phi)) + j\Delta \sin(\phi).$$

(23)

This means that $\theta_{12} = \theta' - \theta$ is given by

$$\theta_{12} = \arctan\left(\frac{\Delta \sin(\phi)}{R + \Delta \cos(\phi)}\right) \approx \frac{\Delta \sin(\phi)}{R}$$

(24)

where the approximation (a) is valid for $R \gg \Delta$. Similarly, the absolute value of the random variable associated to an element of $x^{(2)}$ is

$$R' = |R + \Delta \exp(j\phi)| \approx R + \Delta \cos(\phi).$$

(25)

When $x^{(1)}$ and $x^{(2)}$ are subjected to bandpass memoryless nonlinearities, we have

$$y^{(1)} = A(R) \exp(j\Theta(R)) \exp(j\theta),$$

(26)

and

$$y^{(2)} \approx A(R + \Delta \cos(\phi)) \exp(j(\Theta(R + \Delta \cos(\phi)) \exp(j(\theta + \theta_{12})).$$

(27)

Let us now express the nonlinearity in the Cartesian form as

$$f(R) = A(R) \cos(\Theta(R)) + jA(R) \sin(\Theta(R)) = f_I(R) + jf_Q(R).$$

(28)

Using a Taylor approximation, the nonlinearity output for $x^{(2)}$ is

$$y^{(2)} \approx ((f_I(R) + f_I(R)\Delta \cos(\phi)) + j(f_Q(R) + f_Q(R)\Delta \cos(\phi)) \exp(j\theta').$$

(29)

For $x^{(1)}$, we can write

$$y^{(1)} = (f_I(R) + jf_Q(R)) \exp(j\theta).$$

(30)

To obtain the Euclidean distance between these two signals, we will start by evaluating the
difference between them.
We have \( y^{(2)} - y^{(1)} \approx \)
\[
(f_1(R) + f_1(R)\Delta \cos(\phi) + j(f_0(R) + f_0(R)\Delta \cos(\phi))) \exp(j\theta_{12} + \theta) - (f_1(R) + j f_0(R))\exp(j\theta)
\]
\[
= (f_1(R) + j f_0(R)) + (f_1(R)\Delta \cos(\phi) + j f_0(R)\Delta \cos(\phi))
\]
\[
\Delta \cos(\phi) - (f_1(R) + j f_0(R))\exp(-j\theta_{12})
\]
\[
- (A(R)\exp(j\Theta(R)))\exp(-j\theta_{12})
\]
\[
= A(R)\exp(j\Theta(R))(1 - \exp(-j\theta_{12}))
\]
\[
+ (f_1(R)\Delta \cos(\phi) + j f_0(R)\Delta \cos(\phi)).
\]

(31)

Recalling that \( \sin(\theta_{12}) \approx \theta_{12} \) and \( \cos(\theta_{12}) \approx 1 \) for low values of \( \theta_{12} \) (say \( \theta_{12} = 1 \)), we can write that

\[
y^{(2)} - y^{(1)} \approx A(R)\exp(j\Theta(R))\int \left( \frac{\Delta \sin(\phi)}{R} \right)
\]
\[
+ (f_1(R)\Delta \cos(\phi) + j f_0(R)\Delta \cos(\phi))
\]
\[
(32)
\]

After some manipulations, if we take the squared absolute value of (32), we may write

\[
|y^{(2)} - y^{(1)}|^2 \approx \Delta^2((A'(R)\cos(\phi))^2 + \frac{(A(R)\sin(\phi))}{R}
\]
\[
+ \Theta'(R)A(R)\cos(\phi))^2),
\]
(33)

and the Euclidean distance is finally given by

\[
E[D^2] = \|y^{(2)} - y^{(1)}\|^2 \approx \sum_{R=0}^{N-1}E\left[|y^{(2)} - y^{(1)}|^2\right]
\]
\[
= N\Delta^2 \sum_{R=0}^{2\pi} \left( A'(R)\cos(\phi) \right)^2 + \frac{(A(R)\sin(\phi))}{R}
\]
\[
+ \Theta'(R)A(R)\cos(\phi))^2)p(R)p(\phi)\,d\phi
\]
\[
= \sum_{R=0}^{2\pi} \left( A'(R)\cos(\phi) \right)^2 \frac{(A(R)\sin(\phi))}{R}
\]
\[
+ \Theta'(R)A^2(R)(1 + \cos(2\phi)) + \Theta^2(R)A^2(R)(1 + \cos(2\phi))
\]
\[
+ \Theta'(R)A^2(R)\sin(2\phi)
\]
\[
= \frac{1}{2} E[A^2(R)] = \int_0^{2\pi} A^2(R)p(R)\,dR,
\]

(34)

where the approximation \( (b) \) is valid for \( N >> 1 \). After make some manipulations such as the separation of the double integral in two independent integrals with respect to \( \phi \) and \( R \), we can define the Euclidean distance between two nonlinearly distorted signals as

\[
E[D^2] \approx \frac{N\Delta^2}{2} \int_0^{2\pi} \left( A'(R)\cos(\phi) \right)^2 \frac{(A(R)\sin(\phi))}{R}
\]
\[
+ \Theta'(R)A^2(R)(1 + \cos(2\phi)) + \Theta^2(R)A^2(R)(1 + \cos(2\phi))
\]
\[
+ \Theta'(R)A^2(R)\sin(2\phi)p(R)p(\phi)\,d\phi, \]"
Fig. 8. Evolution of the average asymptotic gain with $k_\theta$.

From the figure, it can be pointed out that the accuracy is higher for large values of $N$, since (22) is based on a Gaussian and Taylor approximations, where the error become neglected when we consider a larger number of in-band subcarriers. Nevertheless, it is important to remark the high potential gains of the optimum receiver, specially for high values of $k_\theta$ or, in other words, when strong nonlinear distortion effects are considered. Due to the extreme complexity of the optimum receiver it is difficult to obtain its performance, even for low values of $N$ and small constellations. However, in the asymptotic region, the BER can be approximated by the following expression

$$P_b \approx \sum_i f_{G_i} Q \left( \frac{2E_b G_i}{N_0} \right),$$

(39)

where $G_i$ a possible value for the asymptotic gain $G$ and $f_{G_i}$ is its relative frequency. The approximate BER considering an ideal AWGN channel, $N = 128$ and $M = 4$ is shown in Fig.

Fig. 9. Approximate BER in an ideal AWGN channel with $N = 128$ and different values of $k_\theta$.

From the figure, it can be noted that the high potential gains shown in Fig. 8 are traduced to high potential gains in the BER performance. At $P_b = 10^{-3}$, the gain relatively to linear CO-OFDM is 1.4 dB for $k_\theta = 0.1$, but can reach $G \approx 4$ dB for $k_\theta = 0.2$, which shows an accentuated performance improvement relatively to the conventional, linear CO-OFDM schemes. When dealing with frequency-selective channels characterized by $H = [H_0 \ H_1 \ \cdots \ H_{M-1}] \in \mathbb{C}^{NM}$, where $H_k$ is the channel frequency response for the $k^{th}$ subcarrier, the Euclidean distance between two nonlinearly distorted CO-OFDM signals is conditioned by the channel realization and given by

$$D^2(H) = \|HY^{(1)} - HY^{(2)}\|^2.$$

(40)

Fig. 10 illustrates average BER performance obtained with (39) but considering the average of several channel realizations, since (40) is dependent on $H$. 

Fig. 10: Average BER performance with (39) considering the average of several channel realizations, since (40) is dependent on $H$. 

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The considered frequency-selective channel has 32 multipath components with uncorrelated Rayleigh fading and we assume that $E[|H_k|^2] = 1$.

![Fig. 10. Approximate BER in a frequency-selective channel with $N = 128$ and different values of $k_\theta$.](image)

From the figure, we note that by making use of the diversity inherent to the nonlinear distortion, the potential gains of the optimum receiver in a frequency-selective channel are even higher than when an ideal AWGN channel is taken into account. At $P_0 = 10^{-3}$, the gain relatively to linear CO-OFDM transmission is around 16 dB for $k_\theta = 0.1$. Moreover, as in the ideal AWGN case, higher values of $k_\theta$ lead to higher gains. Naturally, when $k_\theta = 0$ there are no gains relatively to a linear CO-OFDM transmission.

V. CONCLUSIONS

In this paper we considered a CO-OFDM transmission impaired by NLPN distortion effects. We show that the distortion term that is inherent to the nonlinear operation can be considered to be useful information if optimum detection are employed. In these conditions there is no BER degradation due to in-band distortion introduced by the nonlinearity and, in fact, there are very high potential asymptotic gains relatively to the linear case that can be explored by reduced-complexity sub-optimal receivers.

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