# Matrix Formulation of transmission of electromagnetic waves in one-dimensional multilayer plasma structure

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***Abstract*-We consider the transmission of electromagnetic waves in one-dimensional multi-layer plasma structure. The wave transmittance is obtained analytically for any number of layers of underdense plasma using the interface and layer matrices. It is found that reflectionless transmission emerges for any number of layers. The band structure characteristics also emerges even when the number of barriers is quite small. The ellipsometric parameters of the structure under consideration are also calculated.**

***Index Terms*- multilayer plasma structure, transmittance, ellipsometric parameters**

## I. INTRODUCTION

Electromagnetic wave propagation in plasmas is a fundamental problem relevant for many applications of space [1] and laboratory plasmas [2]. Therefore, the study of propagation of electromagnetic waves through plasma structures has attracted many physicists and engineers. Recently, much work has been done to investigate the transmission, reflection, and absorption of electromagnetic waves from plasma structures [3-5]. Among these efforts, there was a special interest in a plasma multilayer structure or plasma photonic crystals. The technical applications of plasma photonic crystals are now expanding widely in plasma lens [6], plasma antenna [7], and plasma stealth aircraft [8]. The reflectionless transmission due to the Fabry-Perot resonance has been observed in plasma multilayer structure which has many potential applications. Some of these are frequency filters [9] and interferometers [10]. Moreover, the plasma multilayer structure shows not only the characteristics of a photonic crystal, but also the characteristics of a plasma. Therefore, it is a novel and dynamic field. It is expected to bring more new physics phenomena and draw more attention and lead to many new applications in the future [3].

In recent years, spectroscopic ellipsometry has been applied successfully as a powerful tool for the study of thin films and surfaces [11-14]. Ellipsometry is an optical measurement technique that characterizes light reflection (or transmission) from samples. The high accuracy of ellipsometry has led to the application of ellipsometry in a wide spectrum of fields such as physics, chemistry, materials, and photographic science, biology, as well as optical, electronic, and biomedical engineering. Several advantages of ellipsometry have been reported, such as high precision and non-destructive measurement of optical parameters and thickness. In ellipsometry, two ellipsometric parameters are usually measured: the relative phase change namely, ($\Delta$) between the two light components and the relative amplitude ratio change $\tan(\psi)$ between the same components.

In a recent paper, the reflectionless transmission of electromagnetic waves in one-dimensional multi-layer plasmas is studied [15]. The wave transmittance is obtained analytically for single-layer underdense plasma as well as for two layers. In that reference, the authors solved Maxwell's equations in individual layers for normal incidence and the study was restricted to a limited number of cells. However, for possible applications, e.g., the filtering of infrared light in optical applications, it is necessary to consider varying angle of incidence and any number of cells. For oblique propagation the split between p-polarized and s-polarized light emerges. Therefore, in the present paper, we treat the transmission and reflection from a structure comprising any number of layers of underdense plasma using the interface and layer matrices for any incidence angle. The ellipsometric parameters of the structure under consideration are also calculated.

## II. THEORY

When treating the reflection and transmission of polarized light at oblique incidence by a multilayer structure between semi-infinite ambient and substrate media, the addition of multiple reflections approach becomes impractical. A more effective approach is the matrix one which based on the fact that the equations that govern the propagation of light are linear and that the continuity of the tangential fields across an interface between two isotropic media can be regarded as a 2 × 2 linear-matrix transformation [11].

We consider a multilayer structure that consists of a stack of 1, 2, 3, …, m parallel, linear, homogeneous, and isotropic layers sandwiched between two semi-infinite ambient (0) and substrate (m + 1) media as shown in Fig. 1. For generality purposes, we assume the $j^{th}$ medium has $d_j$ and $n_j$ as a thickness and a refractive index respectively. The $j^{th}$ interface located at $z_j$ separates the two media of refractive indices $n_j$ and $n_{j+1}$. An incident monochromatic plane wave in medium 0 (the ambient) generates a resultant reflected plane wave in the same medium and a resultant transmitted plane wave in medium m+1 (the substrate). The total field inside any layer consists of two plane waves: a forward-traveling plane wave denoted by (+), and a backward -traveling one denoted by (-). Then

$$E(z) = \begin{pmatrix} E^{+}(z) \\ E^{-}(z) \end{pmatrix}, \qquad (1)$$

Considering the fields between the two parallel planes at z' and z'', one can write

$$E(z') = M\, E(z''), \qquad (2)$$

where $M$ is the transformation matrix between the planes at z' and z''.
By choosing z' and z'' to lie immediately on opposite sides of an interface, located at $z_j$ between layers j and j+1, Eq. (1) becomes

$$E^{a}(z_j - \varepsilon) = [r_j^{a}]\; E^{a}(z_j + \varepsilon), \qquad (3)$$

where $\varepsilon$ is an infinitely small distance, α = p or s indicating the state of polarization, and $[r_j^{a}]$ is called the interface matrix which is given by

$$[r_j^{a}] = \frac{1}{t^{a}_{j,j+1}} \begin{bmatrix} 1 & r^{a}_{j,j+1} \\ r^{a}_{j,j+1} & 1 \end{bmatrix}, \qquad (4)$$

where $r_{j,j+1}$ and $t_{j,j+1}$ are Fresnel reflection and transmission coefficients at the $j,j+1$ interface.
On the other hand, if z' and z'' are chosen inside the $j^{th}$ layer at its boundaries, Eq. (3) becomes

$$E^{a}(z_{j-1} + \varepsilon) = [f_j]\, E^{a}(z_j - \varepsilon), \qquad (5)$$

where $[f_j]$ is called the layer matrix which is given by

$$[f_j] = \begin{pmatrix} e^{i j_j} & 0 \\ 0 & e^{-i j_j} \end{pmatrix}, \qquad (6)$$

where $j_j = k\, n_j \cos q_j d_j$ with $k = w/c$ is the free space wave number and $\theta_j$ is the refraction angle in the $j^{th}$ layer.

The M-matrix for the whole structure can be expressed as a product of the interface and layer matrices that describe the effects of the individual interfaces and layers of the entire stratified structure, taken in proper order, as follows:

$$[M_N^{a}] = [f_1][r_1^{a}][f_2][r_2^{a}] \ldots [f_N][r_N^{a}], \qquad (7)$$

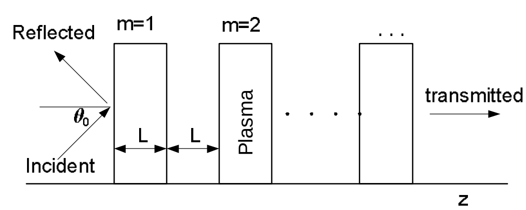


Fig. 1. A Structure of multilayer plasma structure.

Fresnel reflection and transmission coefficients for s and p polarizations are given by

$$r^{s}_{j,j+1} = \frac{n_j \cos q_j - n_{j+1} \cos q_{j+1}}{n_j \cos q_j + n_{j+1} \cos q_{j+1}}, \quad (8)$$

$$t^{s}_{j,j+1} = \frac{2 n_j \cos q_j}{n_j \cos q_j + n_{j+1} \cos q_{j+1}}, \quad (9)$$

$$r^{p}_{j,j+1} = \frac{n_{j+1} \cos q_j - n_j \cos q_{j+1}}{n_{j+1} \cos q_j + n_j \cos q_{j+1}}, \quad (10)$$

$$t^{p}_{j,j+1} = \frac{2 n_j \cos q_j}{n_{j+1} \cos q_j + n_j \cos q_{j+1}}, \quad (11)$$

For the last interface we have $E^{-}_{N} = 0$, so that the reflection and transmission coefficients of the whole system are given by

$$\left.\begin{aligned} r^{a}_{N} &= \frac{\mathrm{E}^{-}_{1}}{\mathrm{E}^{+}_{1}} = \frac{(M^{a}_{N})_{21}}{(M^{a}_{N})_{11}} \\ t^{a}_{N} &= \frac{\mathrm{E}^{+}_{\mathrm{N}}}{\mathrm{E}^{+}_{1}} = \frac{1}{(M^{a}_{N})_{11}} \end{aligned}\right\}, \quad (12)$$

where $(M^{a}_{N})_{ij}$ are the M-matrix elements.

The reflectance of the structure is given by

$$R = \frac{\left|r^{p}_{N}\right|^2 + \left|r^{s}_{N}\right|^2}{2}, \quad (13)$$

whereas the transmittance is given by $T = 1 - R$ provided that no absorbance is present.

The ellipsometric results are usually presented in terms of two parameters $\psi$ and $\Delta$. The fundamental equation in ellipsometry is written in terms of $\psi$ and $\Delta$ as [12-14]

$$r = \tan(y)\, e^{iD} = \frac{r^{p}_{N}}{r^{s}_{N}}, \quad (14)$$

## III. Results and discussion

We consider a structure consisting of alternating layers of two materials. One material is air and the other is plasma of refractive index

$$n(w) = \sqrt{1 - \left(\frac{w_p}{w}\right)^2}, \quad (15)$$

where $\omega$ is the angular frequency of the incident light and $\omega_p$ is the plasma frequency. Here we treat the case of underdense plasma where $w > w_p$ implying that $n < 1$. All layers are assumed to have equal thickness L.

We first consider the special case of normal incidence ($\theta_0 = 0$) and a single-layer plasma ($m = 1$). In Fig. 2, the transmittance $T$ as a function of $(w_p/w)^2$ for $kL$ = 1, 6, and 10 is plotted. As can be seen from the figure, the reflectionless transmission ($T$ = 1) can occur for $kL$ = 6 and 10, which is well-known in optics as the Fabry-Perot resonance. The number of the resonant frequencies that corresponds to reflectionless transmission increases with the increase in the width of the plasma layer. This feature is very obvious in Fig. 3 in which we consider kL = 100.

We now study the transmittance dependence on the number of cells. Figure 4 shows the transmittance of a normally incident wave as a function of $(w_p/w)^2$ for different number of plasma layers. Many interesting features can be observed in the figure. First, the structure is highly transmittive for $(w_p/w)^2 <$ 0.6, especially for m = 2, 4, 8, 10, and 20. Second, the appearance of energy bands in which T is close to unity for large m represents the most important feature of these graphs. These are precursors of the band structure characteristic usually obtained for periodic potentials in quantum mechanics [16,17]. Third, it is interesting that they appear even for a relatively small number of cells. Fourth, each band contains a number of ripples with valleys which progressively narrow as m increases. It is worth to mention that it looks reasonable to assume that each interval of the allowed transmission converges to the transmission bands with T = 1 in the limit of $N \to \infty$. However, this is not the case. As seen in figure 4, the transmission coefficient $T$ rapidly oscillates for large $N$. These oscillations do not disappear in the limit $N \to \infty$ [18].

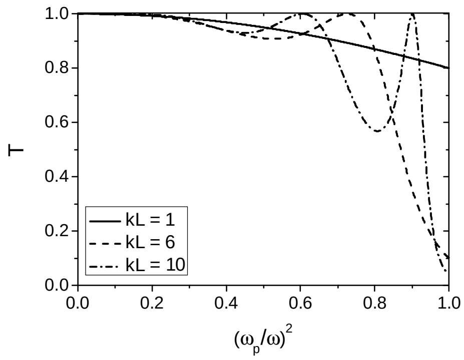


Fig. 2.Transmittance T as a function of $(\omega_p/\omega)^2$ for single-layer plasma for $\theta_0$ = 0 (normal incidence), kL = 1, 6, and 10.

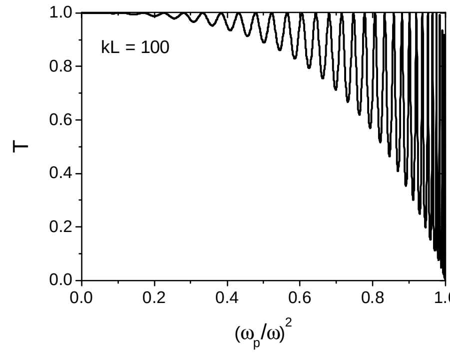


Fig. 3. Transmittance T as a function of $(\omega_p/\omega)^2$ for single-layer plasma for $\theta_0$ = 0 (normal incidence) and kL = 100.

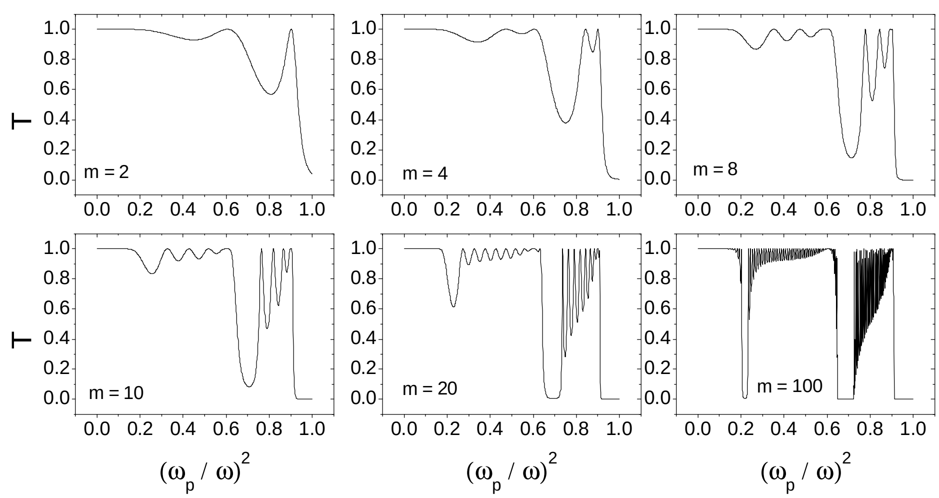


Fig. 4. Transmittance T as a function of $(\omega_p/\omega)^2$ for multilayer plasma structure. We consider $\theta_0$ = 0 (normal incidence), kL = 10 and m = 2, 4, 8 10, 20, and 100.

To investigate a more general case, we consider in Figs. 5 and 6 oblique incidence for m = 2 and m = 6, respectively. The incidence angle has a great impact on the transmittance curves. Figure 5 reveals the structure is highly transmittive for the cases: $(w_p/w)^2 < 1$ for $\theta_0$ = 0, $(w_p/w)^2 < 0.8$ for $\theta_0 = 25^o$, and $(w_p/w)^2 < 0.4$ for $\theta_0 = 50^o$. Therefore, the structure becomes reflective more than transmittive for high angles of incidence. The same feature can be observed in Fig. 6. This feature is simply attributed to the total internal reflection phenomenon. For oblique incidence, total internal reflection takes place as light is traveling from a denser medium (air) to a rarer medium (plasma with refractive index < 1). For example, if we consider the case of $\theta_0 = 50^o$ ($\sin\theta_0 = 0.766$) in Fig. 6, this angle would be a critical angle for $(w_p/w)^2 = 0.413$. Thus for $(w_p/w)^2 > 0.413$, we obtain total internal reflection and the structure becomes highly reflective.

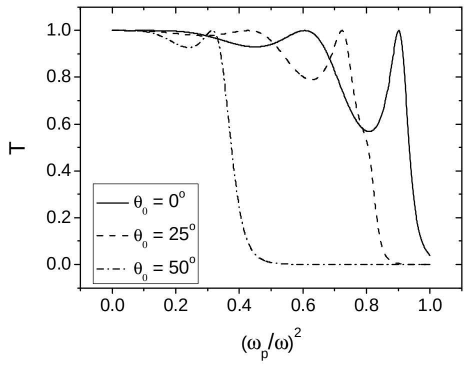


Fig. 5.Transmittance T as a function of $(\omega_p/\omega)^2$ for multilayer plasma structure for different angles of incidence. We consider kL = 10 and m = 2.

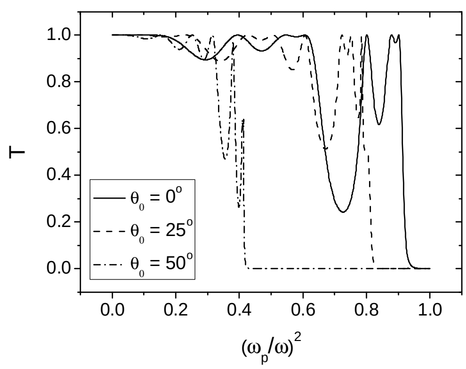


Fig. 6. Transmittance T as a function of $(\omega_p/\omega)^2$ for multilayer plasma structure for different angles of incidence. We consider kL = 10 and m = 6.

As mentioned in the introduction section, the spectroscopic ellipsometry is nowadays a standard optical technique for studying the optical properties of materials and for thin film characterization. An ellipsometric measurement allows one to determine two angles namely, $\Delta$ and $\psi$ which are given by

$$\left.\begin{aligned} D &= d_p - d_s \\ \tan y &= \frac{\left|r_N^p\right|}{\left|r_N^s\right|} \end{aligned}\right\}, \tag{16}$$

where $d_p$ and $d_s$ are the phase changes for the $p$ and $s$ components of light and $r_N^p$ and $r_N^s$ are the complex Fresnel reflection coefficients for the $p$ and $s$ components which may be written as

$$\left.\begin{aligned} r_N^p &= r_p e^{id_p} \\ r_N^s &= r_s e^{id_s} \end{aligned}\right\}, \tag{17}$$

Once $\psi$ and $\Delta$ are determined during a measurement at a given wavelength one can invert Fresnel equations to extract the optical parameters of a bulk sample. For a substrate/thin film/ambient structure one must at least measure at several wavelengths and require that the same value of the film thickness $d$ gives the best fit for the real and imaginary parts of the film index $n$ and $k$ at each wavelength, or, better yet, measure spectra at at least two different angles of incidence. If the film is known to have $k$ = 0, then $n$ and $d$ can be found from one measurement.

Figures 7 and 8 show the ellipsometric parameters of the plasma structure under consideration for m = 2 at two different angles of incidence. As can be seen from Fig. 7, $\psi = 0$ at $(w_p/w)^2 =$ 0.783 and $\theta_0$ = 25°. This corresponds to Brewster angle at which $r_N^p$ goes to zero. Also, for $(w_p/w)^2 >$ 0.903, $\psi$ = 45° for both angles of incidence corresponding to $\left|r_N^p\right| = \left|r_N^s\right| = 1$. In the case of total internal reflection, the complex Fresnel reflection coefficients are given by $r_N^p = e^{id_p}$ and $r_N^s = e^{id_s}$ [19]. The amplitudes of Fresnel reflection coefficients is unity for both polarizations indicating that the ellipsometric parameter $\psi$ = 45°.

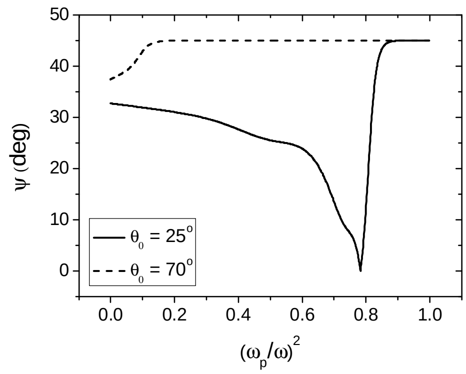


Fig. 7. The ellipsometric parameter ψ as a function of $(\omega_p/\omega)^2$ for multilayer plasma structure for kL = 10, and m = 2.

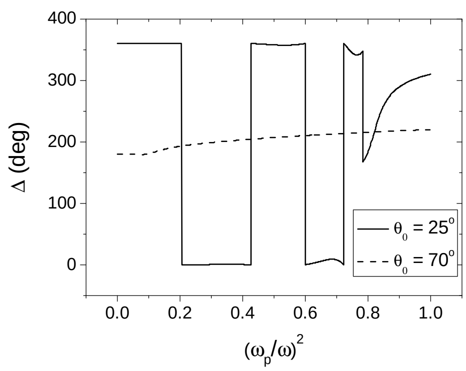


Fig. 8. The ellipsometric parameter Δ as a function of $(\omega_p/\omega)^2$ for multilayer plasma structure for kL = 10, and m = 2.

## IV. CONCLUSION

The transmission of electromagnetic waves in one-dimensional multi-layer plasma structure has been investigated for any number of layers of underdense plasma using the interface and layer matrices. We found that the reflectionless transmission can be possible for any number of layers. The energy band structure has been obtained especially for large number of cells. We have also determined the ellipsometric parameters of the structure from which all the structure parameters can be obtained.

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