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Analysis of Multi-band Rectangular Microstrip Antennas

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Abstract: **Dual and triple band rectangular microstrip antenna is realized by cutting slots at an appropriate position inside the patch. The slot is said to introduce a mode near the fundamental mode resonance frequency of the patch when its length either equals quarter wave or half wave in length. However, this simpler approximation does not give accurate result while designing them at given frequencies. In this paper, first the comparison of slot frequency calculated by using their half wave or quarter wavelength approximation against the simulated results is presented. This approximation does not give closer results for most of the lengths. Hence an analysis to study the effects of slot (half U-slot and rectangular slot) in dual and triple band rectangular microstrip antennas is presented. Through the analysis it was observed that the slot reduces the resonance frequencies of orthogonal and higher order TM_{01} and TM_{11} modes of the rectangular patch and along with TM_{10} mode, yields dual and triple band response. By studying surface current distributions at various modified modes, a formulation of resonant length at them is proposed. The frequencies calculated using them agrees closely with the simulated and measured results.**

Index Terms: **Dual band Microstrip Antenna, Half U-slot, Rectangular slot, Rectangular Microstrip Antenna, Triple Band Microstrip Antenna**

I. INTRODUCTION

More commonly dual and triple band microstrip antenna (MSA) is realized by cutting slots like, U-slot, half U-slot, rectangular slot and step slot at an appropriate position inside the patch [1 – 8]. The slot is said to introduce a mode near the fundamental mode of the patch, to yield multi-band response. In most of the reported slot cut designs, while designing at desired frequencies, slot length is taken to be either half wave or quarter wave in length. However this simpler approximation does not give closer result. The equivalent circuit model at the patch and slot modes in dual band slot cut MSA is reported [9]. However an insight into the effect of slot and formulation of slot length for the desired slot frequency is not reported in the available literature. Recently an analysis of U-slot cut dual band rectangular MSA (RMSA) is reported [10]. The approximate equations for slot modes in terms of patch and slot dimension were given. However a clear description of the modes at the individual frequencies and the comparison of the calculated frequencies using the proposed formulations against simulated and measured results were not given. An analysis of dual band slot cut circular MSA is reported [11]. It was observed that the slot does not introduce any additional mode but reduces the resonance frequency of second order TM_{21} mode of the circular patch and along with the fundamental TM_{11} mode, yields dual band response.

In this paper, dual and triple band configurations of slot cut rectangular MSA (RMSA) are discussed. First by using quarter wave or half wave length approximations, a comparison of calculated frequencies against the simulated frequencies obtained using IE3D software is presented [12]. For different slot lengths, this simpler approximation does not give closer results. Therefore an analysis to study the effects of slots on dual and triple band response in RMSAs is presented. The resonance curve plots, surface current distributions and radiation pattern plots for different slot lengths were studied. In dual band RMSAs, slot reduces the resonance frequency of orthogonal TM_{01} mode of the patch and along with TM_{10} mode yields dual frequency response. The slot also reduces TM_{11} mode resonance frequency of the RMSA. Further when an additional slot is cut it reduces higher order TM_{11} mode frequency of the slotted rectangular patch and along with TM_{10} and TM_{01} modes, yields triple frequency response. The radiation pattern at TM_{10} and TM_{01} modes is in the broadside direction with orthogonal directions of principle planes whereas the pattern at TM_{11} mode is conical, i.e. maximum in the end-fire direction. In dual band RMSA, at modified TM_{01} mode, contribution of surface currents along patch length increases with slot length. This leads to the radiation pattern with higher cross-polarization levels. At TM_{11} mode, with two slots the contribution of surface currents along the patch length increases which gives broadside radiation pattern at the third frequency. Further by studying surface current distributions at modified TM_{10}, TM_{01} and TM_{11} modes, the formulation of resonant length in terms of slot and patch dimensions at dual and triple frequencies is proposed. The frequencies calculated using proposed formulations agrees closely with simulated results obtained using IE3D software and the measured results. The proposed analysis is carried out on glass epoxy substrate (ε_r = 4.3, h = 0.16 cm and tan δ = 0.02). Since these configurations are analyzed on lossy substrates, antenna gain is less than 0 dBi. To improve upon gain, these configurations can be analyzed using their suspended configurations. The proposed analysis will help in understanding the functioning of slot cut multi-band antennas.

II. DUAL AND TRIPLE BAND RMSAs

Dual band rectangular and half U-slot cut RMSAs are shown in Fig. 1(a – c). For to have TM_{10} mode frequency around 900 MHz, RMSA length (L) is taken to be 8 cm. For smaller patch size, its width (W) is taken equal to 5 cm. The quarter wavelength for this patch length is 4 cm. In reported literature on slot cut MSAs, inner half U-slot length and rectangular slot length (slot cut on the patch edge) is taken equal to quarter wave in length. For rectangular slot cut inside the patch, its length is taken equal to be half wave in length. The frequencies calculated using these approximation for different slot lengths (L_h or l) and their positions (Y and L_v) against the simulated frequencies are shown in Fig. 1(d – f). Using this simpler approximation, for the entire slot length range, larger % error is observed.

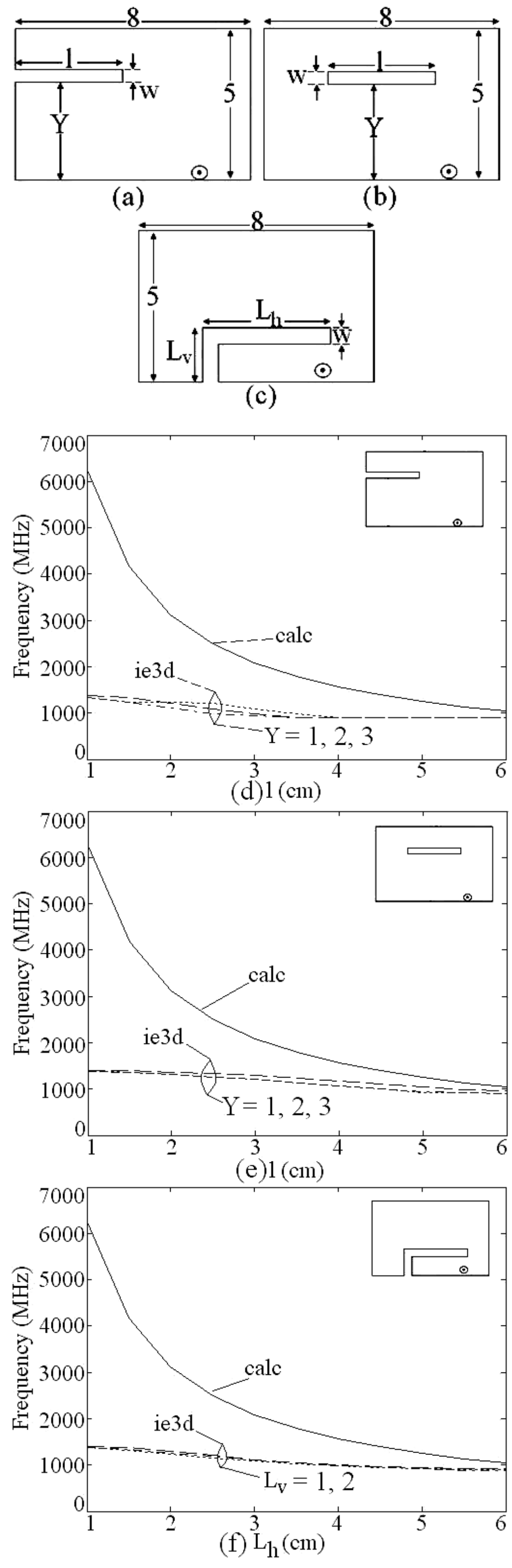

Fig. 1 Dual band (a, b) rectangular slot and (c) half U-slot cut RMSAs and its (d – f) frequency and % error plots

For dual band RMSAs shown in Fig. 1(a, b), dual frequency response is realized for two different slot lengths which are not equal to quarter wave length as shown in Fig. 2(a, b). The fabricated prototype of these configurations is shown in Fig. 3(a, b). In quarter wave or half wave length approximation, slot frequency is independent of slot position (Y). However for MSA shown in Fig. 1(a), for l = 2.0 cm, frequency due to slot changes by 100 MHz, when 'Y' is varied from 1 to 2 cm, as shown in Fig. 3(c). Similar results are observed in triple band RMSAs when another slot is cut inside the patch. Thus it can be inferred from above study that half or quarter wave length approximation does not give closer results. Therefore to understand the functioning of dual and triple band RMSAs, a detail analysis of slot cut antennas is needed.

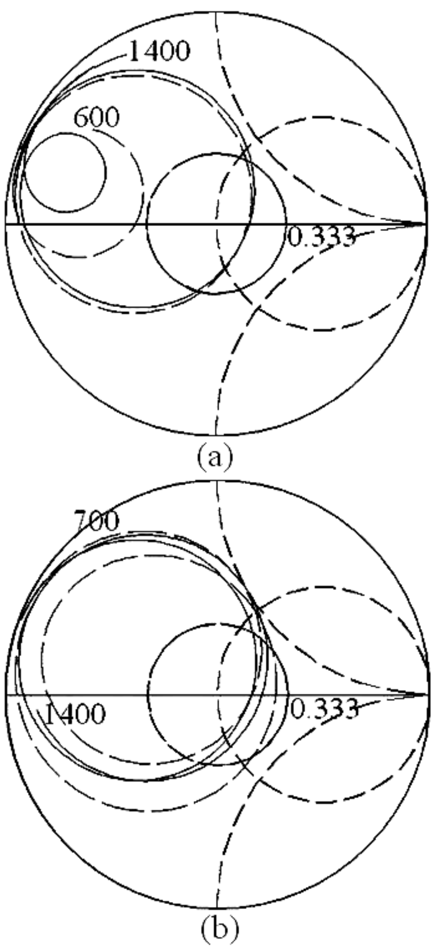

Fig. 2 Measured input impedance plots for dual band RMSA shown in (a) Fig. 1(a), (——) l = 2 cm, (– – –) l = 5 cm, and (b) Fig. 1(b), (——) l = 2 cm, (– – –) l = 5 cm

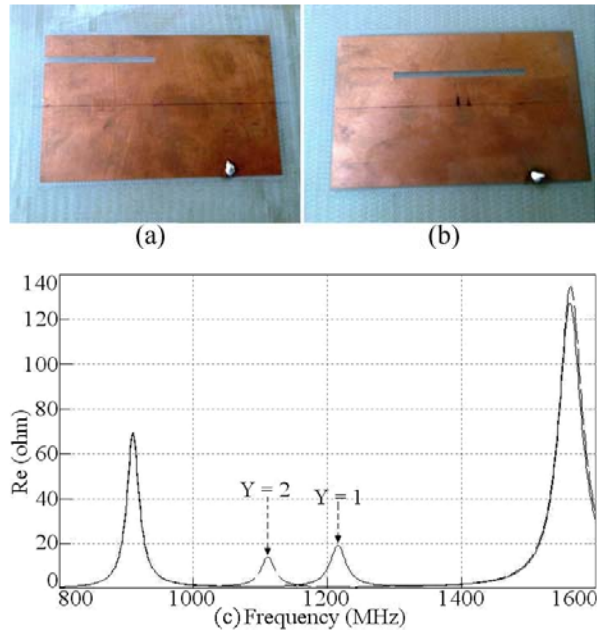

Fig. 3 (a, b) Fabricated prototype of dual band RMSAs and (c) resonance curve plot for different Y for dual band RMSA shown in Fig. 1(a)

III. ANALYSIS OF DUAL AND TRIPLE BAND SLOT CUT RMSAs

The resonance frequencies of fundamental and higher order modes of equivalent RMSA in 700 to 3000 MHz range, calculated using its resonance frequency equation are, f_{TM10} = 887 MHz, f_{TM01} = 1404 MHz, f_{TM11} = 1661 MHz, f_{TM20} = 1775 MHz, f_{TM02} = 2808 MHz. For the feed location shown in Fig. 1(a), RMSA is simulated using IE3D and its resonance curve plot is shown in Fig. 4(a). The peaks due to TM_{10}, TM_{01} and TM_{11} modes are present and surface current distributions at them are shown in Fig. 4(b – d). At TM_{10} mode, one half wavelength variations in surface current is present along patch length. At TM_{01} mode, currents show one half wave length variations along the patch width, which gives broadside radiation pattern with E-plane aligned along $\Phi = 90^0$, as shown in Fig. 4(e). At TM_{11} mode, currents show one half wavelength variations along patch length as well as width. This leads to conical radiation pattern, as shown in Fig. 4(f). To realize dual band RMSA (Fig. 1(a)), a slot of length 'l' and width (w) 0.2 cm is cut at distance 'Y' from the feed point axis. For given 'Y', slot length was increased in steps of 0.5 cm and resonance curve plot, surface current distribution and radiation pattern plots for each of the lengths were studied. The resonance curve plots for this length variation are shown in Fig. 4(a).

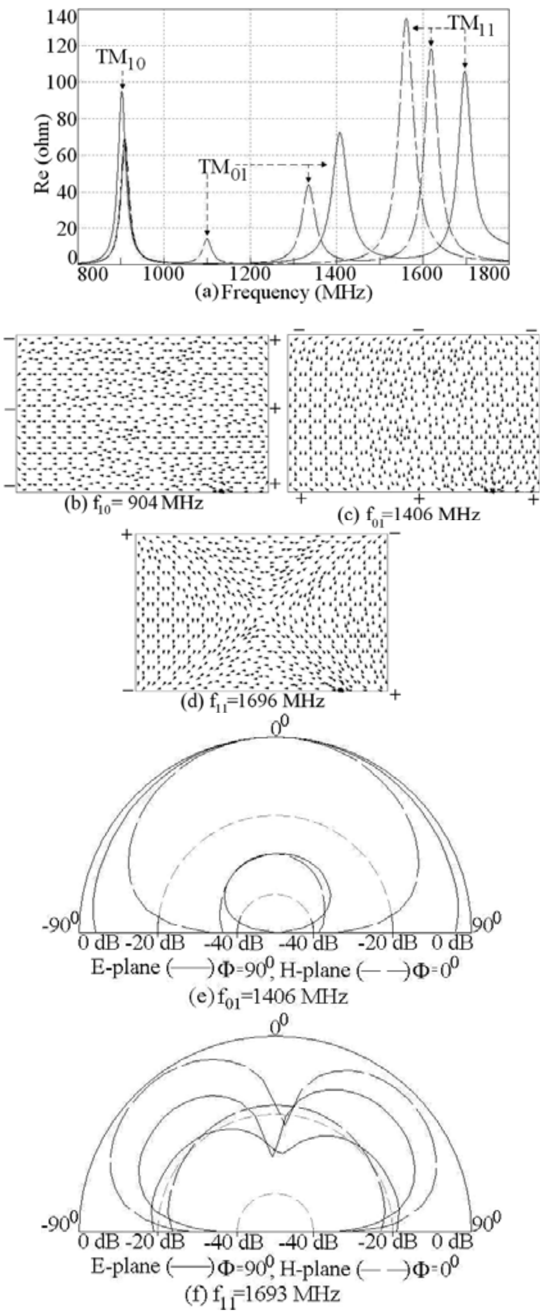

Fig. 4 (a) Resonance curve plots for (—) RMSA, (— — —) l = 1.0 cm, (— - —) l = 2.0 cm, (b – d) surface current distribution and (e, f) radiation pattern at various modes for RMSA

An increase in slot length is orthogonal to the surface currents at TM_{01} mode hence its frequency reduces. At TM_{11} mode, surface currents along patch width are perturbed which reduces its frequency. The surface current distribution and radiation pattern plots at modified TM_{01} mode for three different slot lengths are shown in Fig. 5(a – f).

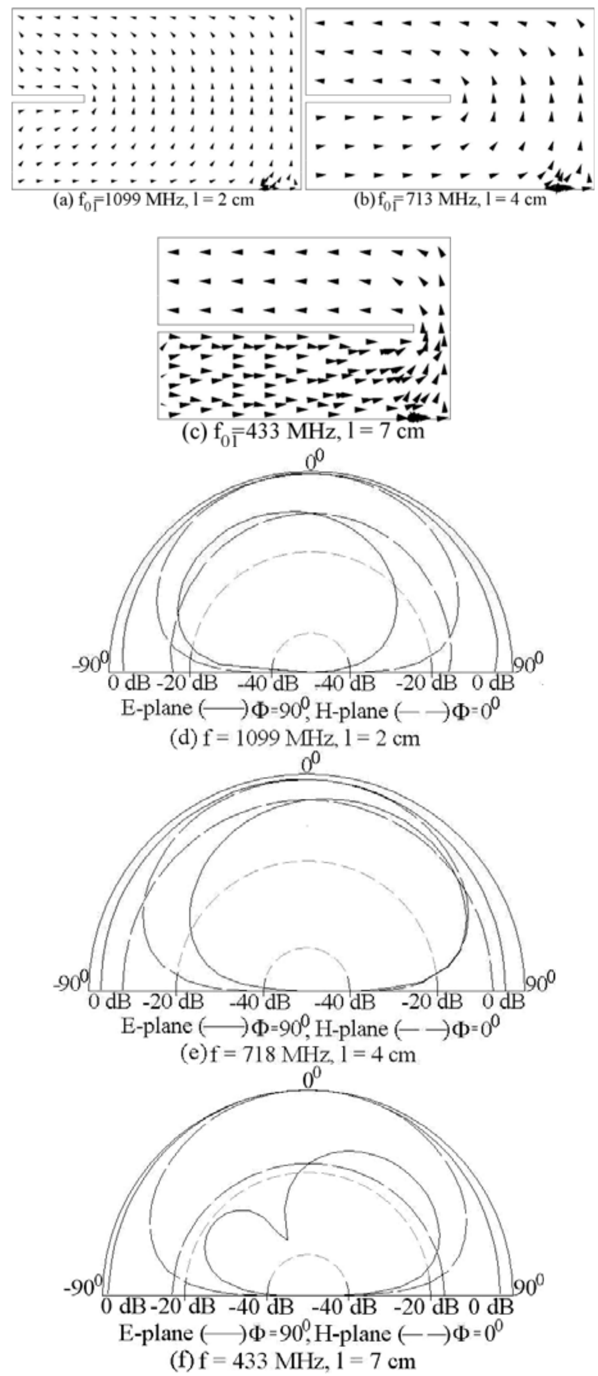

Fig. 5 (a – c) Surface current distribution and (d – f) radiation pattern for different slot lengths at TM_{01} mode for dual band RMSA shown in Fig. 1(a)

With an increase in slot length contribution of surface currents along patch length increases which increases the cross polar levels. For larger slot lengths (l > 6 cm), fields due to equal and oppositely directed currents along patch length cancels in the far field direction which results in lower cross polar levels. The currents directed along patch width leads to broadside radiation pattern. For the current distributions shown in Fig. 5, Y is taken to be 2.5 cm. When Y is increased, for l > 6 cm, the pattern shows higher cross polar levels as radiation due to surface currents along patch length does not cancel in broadside direction. Further dual band response is realized when the spacing of modified TM_{01} mode is optimized with respect to TM_{10} mode. Similar analysis is carried out for rectangular slot (slot cut inside the patch) and half U-slot cut RMSAs and their resonance curve plots and surface current distributions are shown in Figs. 6(a – c) and 7(a – c).

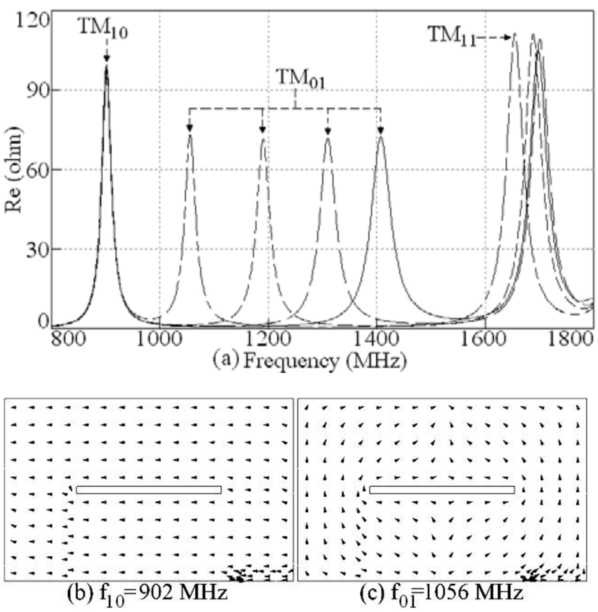

Fig. 6 (a) Resonance curve plots for rectangular slot cut RMSA shown in Fig. 1(b), (——) l = 0, (— —) l = 2, (- - -) l = 3, (— - —) l = 4, its (b, c) surface current distribution at dual frequencies, l = 4

In both of these RMSAs, orthogonal TM_{01} mode frequency reduces with the slot length and comes closer to TM_{10} mode frequency and realizes dual frequency response. The radiation pattern at modified TM_{01} mode shows higher cross polarization levels due to the horizontal surface current components as shown in Fig. 7(d, e). In both of these MSAs, TM_{11} mode frequency reduces with an increase in slot length.

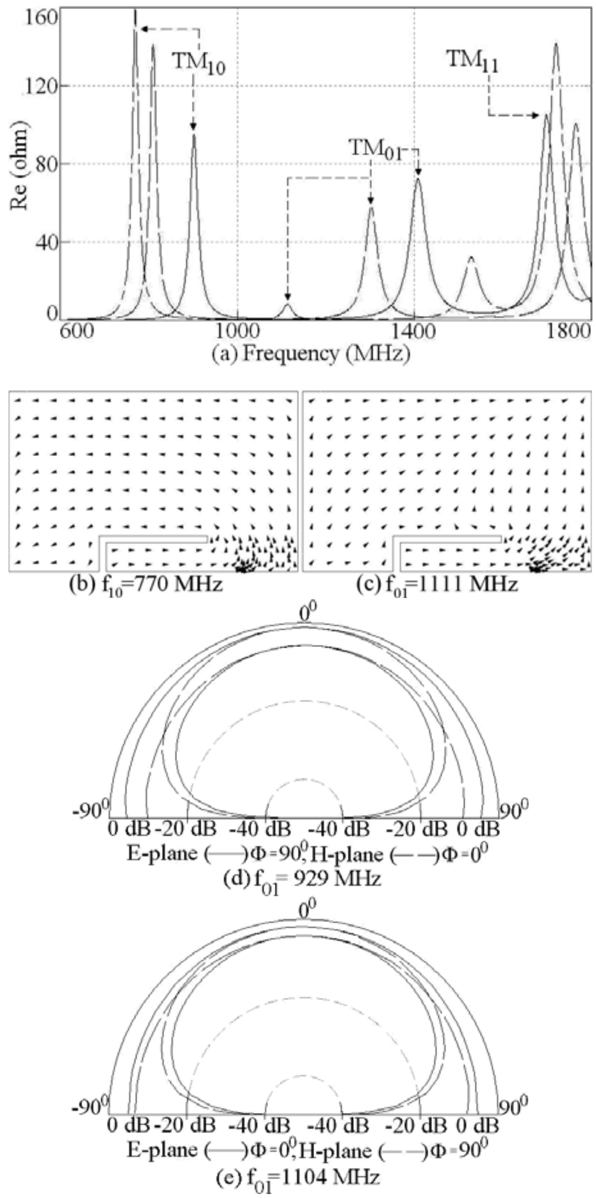

Fig. 7 (a) resonance curve plot for half U-slot cut RMSA, (——) RMSA, (— —) L_h = 2.0, L_v = 1.0, (— - —) L_h = 3.0, L_v = 1.0, and its (b, c) surface current distribution at dual frequencies, L_h = 3.0, L_v = 1.0, radiation pattern for (d) rectangular slot (l = 5 cm) and (e) half U-slot cut (L_h = 3.0) RMSA at TM_{01} mode

The triple band RMSA is realized by cutting an additional slot inside slot cut RMSA and two variations of half U-slot cut triple band RMSA are shown in Fig. 8(a, b). In both the triple band RMSAs, resonance curve plots, surface current distributions and simulated radiation pattern plots are studied for different values of horizontal and half U-slot lengths and they are shown in Fig. 8(c – f).

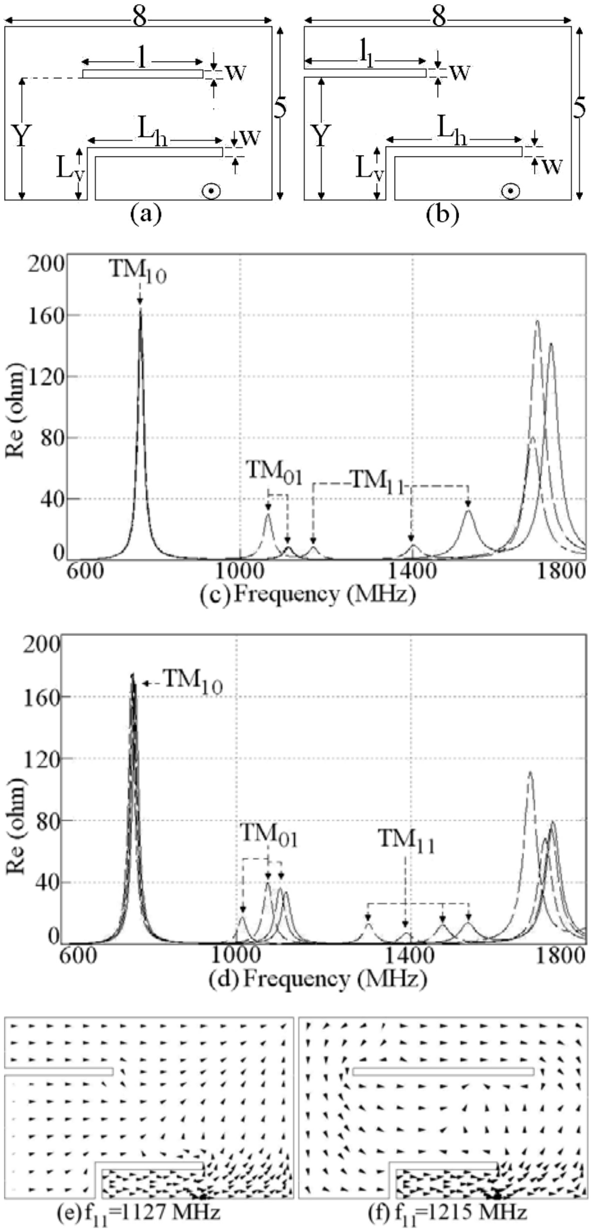

Fig. 8 (a, b) Triple band half U-slot and rectangular slot cut RMSAs and their resonance curve plots (c) (——) L_h = 3.0, L_v = 1.0, l = 0, (– – –) L_h = 3.0, L_v = 1.0, l = 1, (— - —) L_h = 3.0, L_v = 1.0, l = 2, (d) (——) L_h = 3.0, L_v = 1.0, l_1 = 0, (— —) L_h = 3.0, L_v = 1.0, l_1 = 2, (– – –) L_h = 3.0, L_v = 1.0, l_1 = 3, (— - —) L_h = 3.0, L_v = 1.0, l_1 = 4, and surface current distribution at modified TM_{11} mode for triple band RMSA, (e) l_1 = 3 cm and (f) l = 5 cm

With an increase in rectangular slot length, TM_{01} and TM_{11} mode frequencies reduce as the slot is orthogonal to the surface currents. The triple frequency response is realized when the spacing of modified TM_{11} frequency is optimized with respect to TM_{10} and TM_{01} mode frequencies. The rectangular slot modifies the current distribution at TM_{11} mode and aligns them along the patch length. Thereby it gives broadside radiation pattern with E-plane aligned along $\Phi = 0^0$ as shown in Fig. 9(a, b). The fabricated prototypes of triple band RMSAs are shown in Fig. 9(c, d).

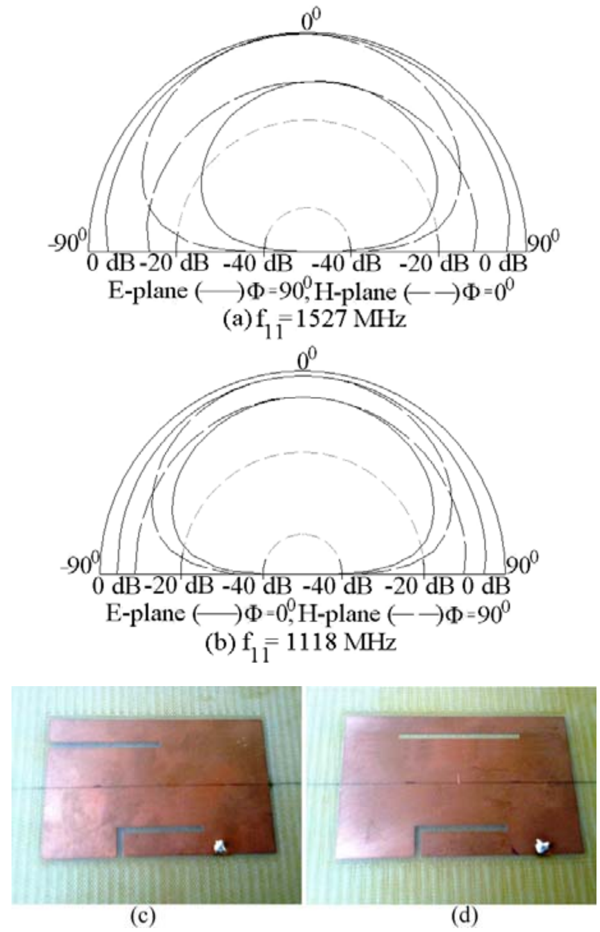

Fig. 9 Radiation pattern at modified TM_{11} mode for triple band RMSA shown in Fig. 8(b) for (a) l_1 = 0 and (b) l_1 = 4 cm, (c, d) fabricated prototypes of triple band RMSA

In the next section by studying the surface current distribution at TM_{10} and modified TM_{01} and TM_{11} modes, formulation of resonant length for dual and triple band RMSAs is proposed.

IV. FORMULATION OF RESONANT LENGTH FOR MULTI-BAND RMSAs

For the dual band RMSA shown in Fig. 1(a), formulation of resonant length at TM_{01} mode is obtained by modifying patch width in terms of slot dimensions as given in equations (1) – (3).

$$W_e = W + 2\Delta l + \left(l/W\right)\sin\left(Y\pi/W\right) \tag{1}$$

$$f_r = c/2W_e\sqrt{\varepsilon_{re}} \tag{2}$$

$$E = \left(\frac{f_r - f_{ie3d}}{f_{ie3d}}\right)x100 \tag{3}$$

To account for fringing field extensions, 'Δl' is added [1]. Along the patch width, surface currents show sinusoidal variation hence sine term is used in equation (1). For different 'Y', resonance frequency is calculated by using equation (2) and % error (E) between calculated and simulated values is obtained by using equation (3). For Y = 1 and 2 cm, they are plotted in Fig. 10(a, b). For different 'Y', a closer agreement between the two results is obtained.

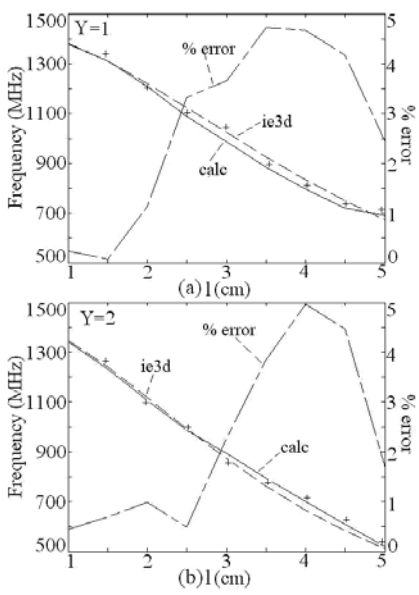

Fig. 10 (a, b) Resonance frequency and % error plots for rectangular slot cut RMSA, (+ + +) measured

Since TM_{10} mode frequency nearly remains constant, its formulation is not proposed. Similarly the formulation for dual band RMSA with slot cut inside the patch, at its modified TM_{01} mode is given by using equation (4). The resonance frequency and % error are calculated by using equations (2) and (3), respectively. For different values of 'Y' they are plotted in Fig. 11(a, b). For the complete slot length range, a closer agreement between calculated and simulated values is obtained.

$$W_e = W + 2\Delta l + 0.67l\left(l/W\right)\sin\left(Y\pi/W\right) \tag{4}$$

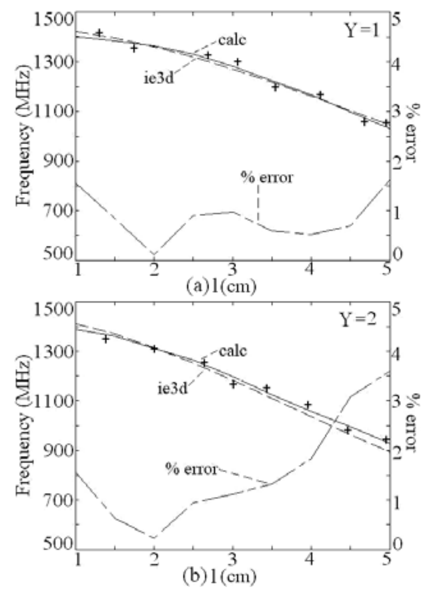

Fig. 11 (a, b) Resonance frequency and % error plots for rectangular slot cut inside RMSA, (+ + +) measured

In dual band half U-slot cut RMSA, TM_{10} mode frequency also reduces due to vertical half U-slot length. Hence formulation at TM_{10} mode is proposed as given in equation (5) which modifies the patch length. The frequency is calculated by using equation (6) and % error between calculated and simulated values is obtained by using equation (3). For different values of L_v, they are plotted in Fig. 12(a, b), which shows closer approximation between the two results.

$$L_e = L + 2\Delta l + 0.5L_v + 2L_h\left(L_h/2L\right) \tag{5}$$

$$f_r = c/2L_e\sqrt{\varepsilon_{re}} \tag{6}$$

The formulation at modified TM_{01} mode is obtained by modifying patch width as given by equations (7) and (8). The frequencies and % error is calculated by using equations (2) and (3) and they are plotted in Fig. 13(a, b). For entire slot length range a closer agreement between the two results is obtained.

$$W_e = W + 2\Delta l + 2AL_h \tag{7}$$

$$A = \left(\frac{L_h}{2W}\right)\sin\left(\frac{\pi\left(W - L_v\right)}{W}\right) \tag{8}$$

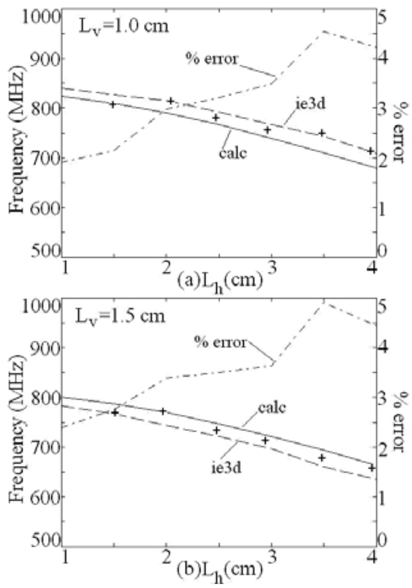

Fig. 12 (a, b) Resonance frequency and % error plots at TM_{10} mode for half U-slot cut RMSA, (+ + +) measured

Similarly the formulation at modified TM_{11} mode is obtained by using equations (9) – (11) and they are plotted in Fig 13(c, d).

$$L_e = L + 2\Delta l + 0.4L_v + 2L_h\left(\frac{L_h}{5L}\right) \tag{9}$$

$$W_e = W + 2\Delta l + 2L_h\left(\frac{L_h}{5W}\right)\sin\left(\frac{\pi\left(W - L_v\right)}{W}\right) \tag{10}$$

$$f_r = \frac{c}{2\sqrt{\varepsilon_{re}}}\sqrt{\left(\frac{1}{L_e^2}\right)+\left(\frac{1}{W_e^2}\right)} \tag{11}$$

At TM_{11} mode, for entire slot length range a closer approximation between two results is obtained. In triple band RMSAs, rectangular slot is parallel to the surface currents at TM_{10} mode. Hence reduction in its frequency is negligible. The slot reduces modified TM_{01} and TM_{11} mode frequencies as it is orthogonal to the surface currents. For half U-slot cut RMSA with rectangular slot cut on the patch edge, formulation at modified TM_{01} mode frequency is obtained by using equations (12) – (14).

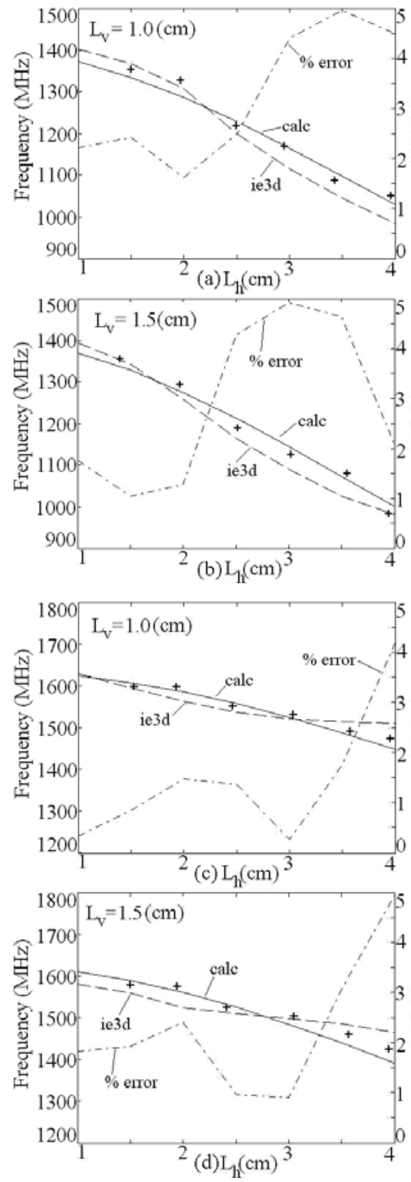

Fig. 13 Resonance frequency and % error plots at (a, b) TM_{01} and (c, d) TM_{11} mode for dual band half U-slot cut RMSA, (+ + +) measured

In these formulations the resonant length at modified TM_{01} or TM_{11} modes in half U-slot RMSA is modified to account for the effects of rectangular slot. The third term in equation (12) accounts for half U-slot whereas fourth term accounts for rectangular slot. The frequency is calculated by using equation (2) and for Y = 2.5 cm, it is plotted in Fig. 14(a). It shows good agreement between calculated, simulated and measured values. Similarly the formulation at TM_{11} mode is obtained by using equations (15) – (19) and the frequency is calculated by using equation (11). For Y = 2.5, resonance frequency and % error plots are shown in Fig. 14(b). It shows close agreement between two results. Similar results are obtained for other values of Y at both the modes.

At TM_{01} mode,

$$W_e = W + 2\Delta l + 2AL_h + 2Bl + 0.4L_v \quad (12)$$

$$A = \frac{L_h}{2.5W} \sin\left(\frac{\pi(W - L_v)}{W}\right) \quad (13)$$

$$B = \frac{1}{2.2W} \sin\left(\frac{\pi Y}{W}\right) \quad (14)$$

At TM_{11} mode,

$$L_e = L + 2\Delta l + 2CL_h + 0.4L_v \quad (15)$$

$$C = \frac{L_h}{5L} \quad (16)$$

$$W_e = W + 2\Delta l + 2DL_h + 2lE \quad (17)$$

$$D = \frac{L_h}{5W} \sin\left(\frac{\pi(W - L_v)}{W}\right) \quad (18)$$

$$E = \frac{1}{W} \sin\left(\frac{\pi(W - Y)}{W}\right) \quad (19)$$

The formulation at modified TM_{01} and TM_{11} modes in half U-slot cut RMSA with rectangular slot cut inside the patch is given by using equations (20) to (27). In these formulations, corresponding resonant length due to half U-slot at respective modes is modified to account for the effect of the rectangular slot. At TM_{01} and TM_{11} modes, frequency is calculated by using equations (2) and (11), respectively and for Y = 2.5 cm, they are plotted in Fig. 15 and 16. For the complete slot length range a closer approximation between the two results is obtained. Similar results were obtained for Y = 3.5 and 1.5 cm.

At TM_{01} mode,

$$W_e = W + 2\Delta l + 2GL_h + 2Hl + 0.5L_v \quad (20)$$

$$G = \frac{L_h}{2.5W} \sin\left(\frac{\pi(W - L_v)}{W}\right) \quad (21)$$

$$H = \frac{1}{10W} \sin\left(\frac{\pi Y}{W}\right) \quad (22)$$

At TM_{11} mode,

$$L_e = L + 2\Delta l + 2JL_h + 0.4L_v \quad (23)$$

$$J = \frac{L_h}{5L} \quad (24)$$

$$W_e = W + 2\Delta l + 2KL_h + 2lM \quad (25)$$

$$K = \frac{L_h}{5W} \sin\left(\frac{\pi(W - L_v)}{W}\right) \quad (26)$$

$$M = \frac{1}{3.5W} \sin\left(\frac{\pi Y}{W}\right) \quad (27)$$

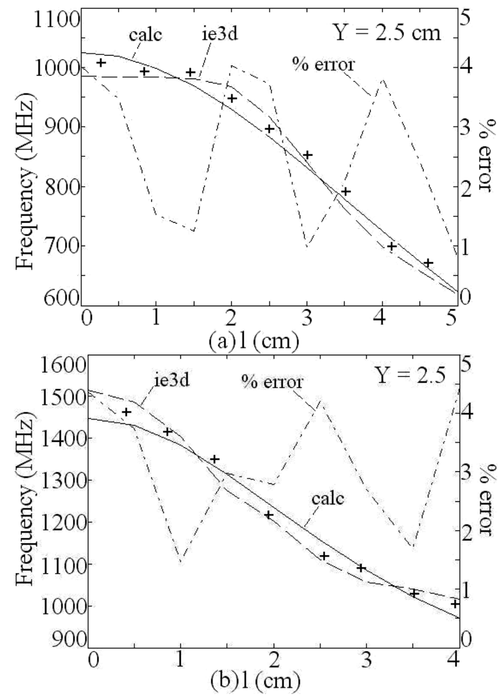

Fig. 14 Resonance frequency and % error plots at (a) TM_{01} and (b) TM_{11} modes for half U-slot and rectangular slot cut RMSA, (+ + +) measured

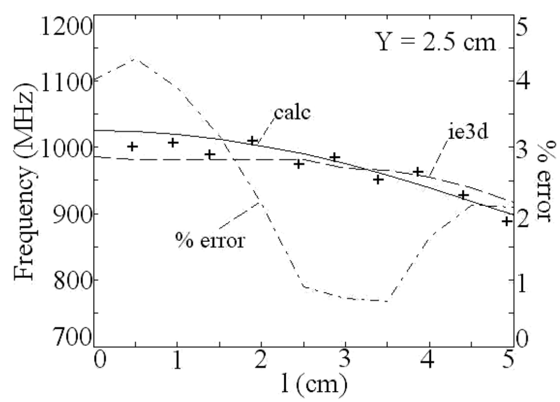

Fig. 15 Resonance frequency and % error plots for half U-slot cut RMSA with rectangular slot cut inside the patch at TM_{01} mode, (+ + +) measured

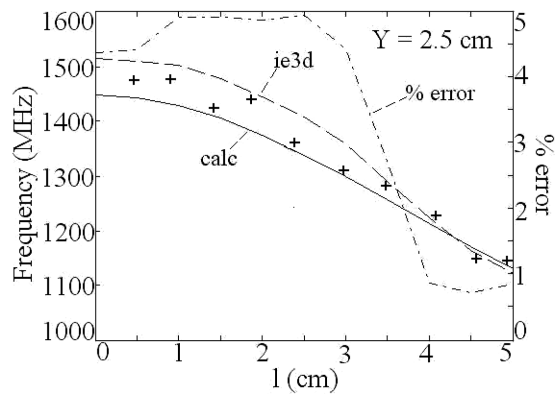

Fig. 16 Resonance frequency and % error plots for half U-slot cut RMSA with rectangular slot cut inside the patch at TM_{11} mode, (+ + +) measured

V. CONCLUSIONs

Dual and triple band slot cut RMSAs are discussed. To design them at given frequencies, half wave or quarter wave length approximation, shows larger error. Hence an analysis to study the effects of slot in dual and triple band RMSAs is presented. The slot does not introduce any additional mode but reduces the resonance frequencies of orthogonal and higher order modes of the patch and along with the fundamental TM_{10} mode yields multi-band response. In dual band RMSAs, the radiation pattern shows higher cross-polar levels due to orthogonal surface current component. In triple band RMSA, slot modifies surface current distributions at all the three modes to realize broadside radiation pattern at the three frequencies. Further by studying surface current distributions at modified TM_{10}, TM_{01} and TM_{11} modes, a formulation in resonant length for dual and triple band RMSAs is proposed. The frequencies calculated using proposed formulations agrees closely with simulated and measured results. Since, the idea was to prove the concept for resonance frequency, and not the radiation efficiency/gain, these configurations are analyzed and validated using low cost lossy glass epoxy substrates. For this substrate the radiation efficiency is very low and thus antenna gain is near to 0 dBi. To improve upon gain, these configurations can be analyzed using suspended configurations or on substrate, like RT-Duroid or air, having very low loss tangent. However, directivity of these antennas is high of the order of 5 to 7 dBi. Thus, normalized radiation patterns will remain unchanged. The proposed formulations can be used for suspended antennas, as they are proposed in terms of slot and patch dimensions and dielectric constant of the substrate. The loss tangent term which mainly accounts for loss, is not included, as it primarily affects the input impedance matching, than the resonance frequency. The proposed study gives an insight into the functioning of slot cut multi-band antennas and proposed formulations can be used to design them at given frequencies.

REFERENCES

[1] R. Garg, P. Bhartia, I. Bahl and A Ittipiboon, *Microstrip Antenna Design Handbook*, Artech House, USA, 2001.

[2] B. Bhartia and I. J. Bahl, *Microstrip Antennas*, USA, 1980.

[3] G. Kumar and K. P. Ray, *Broadband Microstrip Antennas*, Artech House, USA, 2003.

[4] J. H. Lu, "Single Feed Dual Frequency Rectangular Microstrip Antenna With Pair Of Step Slots", *Electronics Letters*, Vol. 35, No. 5, pp. 354 – 355, 4th March 1999.

[5] A. E. Daniel, Tunable dual band Rectangular Microstrip antennas and their Arrays, Ph.D. Thesis, I. I. T. Bombay, 2006

[6] A. A. Deshmukh, and K. P. Ray, "Multi-band Rectangular Microstrip Antennas", *Microwave & Opti. Tech. Letters*, Vol. 49, No. 11, pp. 2757 – 2761, Nov. 2007.

[7] K. F. Lee, K. M. Luk, K. M. Mak, and S. L. S. Yang, "On the use of U-slots in the design of Dual and Triple band Patch Antennas", *IEEE Antennas and Propagation Magazine*, Vol. 53, No. 3, pp. 60 – 74, June 2011

[8] K. F. Lee, S. L. S. Yang, and A. A. Kishk, "Dual and Multiband U-slot patch Antennas", *IEEE Antennas and wireless Propagation Letters*, Vol. 7, pp. 645 – 647, 2008

[9] J. A. Ansari, S. K. Dubey and P. Singh, "Analysis of U-slot loaded Patch for Dual band operation", *International Journal of Microwave and Optical Technology*, Vol. 3, No. 2, pp. 80 – 84, April 2008

[10] S. G. Weigand, H. Huff, K. H. Pan, and J. T. Bernhard, "Analysis and design of Broadband Single Layer Rectangular U-slot Microstrip Patch Antenna", *IEEE Transactions on Antennas & Propagation*, Vol. 51, No. 3, pp. 457 – 468, 2003.

[11] A. A. Deshmukh, K. P. Ray, T. Sonwadkar, S. Varawalla and R. Kataria "Formulation of Resonance Frequency for Multi-Band Circular Microstrip Antennas", *International Journal of Microwave and Optical Technology*, Vol. 5, No. 5, September 2010, pp. 248 – 256.

[12] IE3D 12.1, Zeland Software, Freemont, USA, 2004.