Negative Refraction in One Dimensional Photonic Crystal without Negative Refractive Index Material

Vipin Kumar*, Kh. S. Singh. S. P. Ojha[1]

Department of Physics, Digember Jain (P.G.) College, Baraut-250611, INDIA
[1]Ex Vice Chancellor, Chaudhary Charan Singh University, Meerut
Tel: +91-1234-262060; Fax: +91-1234-262060; E-mail: vrpcommon@yahoo.com

Abstract: **Negative refraction occurs in Meta-materials which have simultaneous negative permittivity ε and permeability μ. In the present communication, it has been shown that photonic crystals (PCs) may also exhibit negative refraction, although they have a periodically modulated positive permittivity ε and permeability μ. We have theoretically studied the negative refraction in one-dimensional (1D) photonic crystals (PCs) consisting of dielectric Na_3AlF_6 with Ge. By using transfer matrix method and Bloch theorem, we have studied the photonic band structure and group velocity, and with the help of group velocity, we have obtained the frequency bands of negative refraction. We found that negative refraction may occur near the low frequency edge of the even band gaps.**

Index Terms: **Photonic crystals, Negative refraction, Group velocity.**

I. INTRODUCTION

It is well known that periodic modulation of the dielectric functions of a medium significantly modifies the spectral properties of the electromagnetic waves. Changing spatial distribution of the dielectric constant, one can effectively control the fundamental optical properties like band structure, reflectivity, group velocity and effective group index etc [1-3]. The optical properties of periodic materials that are transparent to electromagnetic (EM) waves can be characterized by an index of refraction which is given by the relation $n = \sqrt{\varepsilon.\mu}$, where ε is the relative dielectric permittivity and μ is the relative permeability of the medium. Generally, ε and μ both are positive for ordinary materials. While ε may be negative for some materials but natural materials with negative μ are not known.

For certain structures, which are called meta-materials, the effective permittivity and permeability, can take negative values. It means that in such materials the effective index of refraction is less than zero. Therefore, in these materials, phase and group velocity of an electromagnetic wave can propagate in opposite directions. Such materials are also known as negative materials. This concept of negative index of refraction was theoretically proposed for the first time by Veselago [4]. Furthermore, light incident from a conventional right-handed material on meta-materials, will bend to the same side as the incident beam and to hold Snell's law, the refraction angle should be negative.

A periodic array of artificial structures, called split ring resonators (SRRs) suggested by Pendry et al. [5], exhibits negative effective μ for frequencies close to the magnetic resonance frequency. Smith et al. [6] experimentally demonstrated that meta-materials can be physically realized by stacking SRR and thin wire structures as arrays of 1D and 2D structured composite meta-materials (CMM). Experimentally the existence of negative refraction in meta-materials is verified for the first time by Shelby et al. [7]. Recently, negative refraction has been subject of considerable interest, which may provide the possibility of a variety of novel applications of very interesting phenomena, like the super-lens effect [8-10]. A negative refractive index occurs in meta-materials that have simultaneous negative permittivity ε and permeability μ, where inhomogeneities are much smaller than the wavelength of the incoming radiation and it has been demonstrated at microwave wavelengths [3, 11]. On the other hand, it has been shown that this interesting phenomenon may also occur in combination of ordinary materials, which are called photonic crystals (PCs) at optical wavelengths [12-14]. The realization of negative refraction and its experimental verification have also been done by Valanju et al. [15]. They have shown that the transmission of energy is possible when the wave comes in a range of frequencies, which combine to form energy packets. A wave having just one frequency component can be bent in the wrong way, but this is irrelevant because real light never has just one frequency. Cubukcu et al. has been the first to demonstrate negative refraction phenomena in two dimensional (2D) photonic crystals in the microwave regime [16].

Further experimental studies have proved that carefully designed photonic crystals are used for obtaining negative refraction at microwave and infrared frequency regimes [17]. The superprism effect is another exciting property arising from photonic crystals [18]. Boedecker and Henkel [19] mentioned that the simple one dimensional Kronig–Penney model provided an exactly solvable example of a photonic crystal with negative refraction.

In the present communication, it is suggested that even a simple one dimensional periodic structure of semiconductor-dielectric materials having photonic band gaps may exhibit negative refraction near the band edges of certain band gaps. It may be noted that our analysis is done in a scale independent manner *i.e.* in terms of normalized parameters and the result can be applied in any range of wavelength with a proper choice of various physical parameters.

II. THEORETICAL ANALYSIS

The Maxwell equation corresponding to TE-polarized electric field of an electromagnetic wave propagating along the x-axis may be written as [20]

$$\frac{d^2E(x)}{dx^2}+\left[\left(\frac{\omega}{c}n_i(x)\right)^2-\beta^2\right]E(x)=0 \qquad (1)$$

with propagation constant $\beta=\frac{\omega}{c}n_i(x)\sin\theta_i$, where θ_i is the angle of incidence with $i=1,2$ for different materials containing the unit cell and c is the velocity of light. Thus, $n_i(x)$ is the periodic refractive index function of the structure given by

$$n_i(x)=\begin{cases} n_1, & 0<x<a \\ n_2, & a<x<d \end{cases} \qquad (2)$$

Here, $n(x)=n(x+d)$, n_1 and n_2 being the refractive indices of the dielectric materials and $d=a+b$ is the period of the lattice with a and b being the widths of layers with low and high refractive index materials respectively. The schematic diagram of periodic structure considered here is shown in Figure 1.

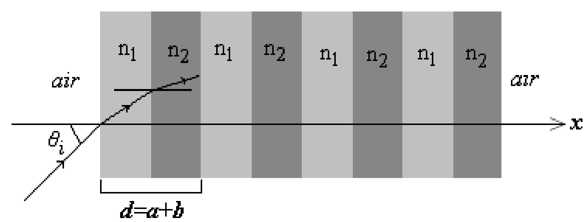

Figure 1: Periodic variation of one-dimensional photonic band gap structure.

The electric field E(x) within each homogenous layer is a combination of right traveling waves and left traveling waves. In each region, the solution of Equation (1) yields the usual plane wave solutions with arbitrary coefficients, which, in mth unit cell can be written as

$$E(x)=\begin{cases} a_m.e^{ik_1x}+b_m.e^{-ik_1x}; & (md-a)<x<md \\ c_m.e^{ik_2x}+d_m.e^{-ik_2x}; & (m-1)d<x<(md-a) \end{cases} \qquad (3)$$

where, $k_i=\left[\left(\frac{n_i.\omega}{c}\right)^2-\beta^2\right]^{\frac{1}{2}}$ with $i=1,2$; a_m, b_m, c_m and d_m are constants and θ_1 and θ_2 are the ray angles in the layers corresponding to refractive indices n_1 & n_2 respectively and these are related with the incident angle θ_0 in air and first unit cell interface given by the relation $n_0\sin\theta_0=n_1\sin\theta_1$.

Now imposing proper boundary conditions on *E(x)* and *dE(x)/dx* at the interfaces i.e. x=(m-1)d and x=(m-1)d+b, we can relate the coefficients of traveling wave inside both region, by the matrix equation [20]

$$\begin{bmatrix} a_{m-1} \\ b_{m-1} \end{bmatrix} = m \begin{bmatrix} a_m \\ b_m \end{bmatrix} \text{ with, } m = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (4)$$

with

$$m_{11} = e^{ik_1 a}\left[\cos(k_2 b) + \frac{i}{2}\left(\gamma + \frac{1}{\gamma}\right)\sin(k_2 b)\right] \quad (5a)$$

$$m_{12} = e^{-ik_1 a}\left[-\frac{i}{2}\left(\gamma - \frac{1}{\gamma}\right)\sin(k_2 b)\right] \quad (5b)$$

$$m_{21} = e^{ik_1 a}\left[\frac{i}{2}\left(\gamma - \frac{1}{\gamma}\right)\sin(k_2 b)\right] \quad (5c)$$

$$m_{22} = e^{-ik_1 a}\left[\cos(k_2 b) - \frac{i}{2}\left(\gamma + \frac{1}{\gamma}\right)\sin(k_2 b)\right] \quad (5d)$$

where $\gamma = k_1/k_2$, for TE polarization.

Now applying Bloch condition, we can derive a relation which gives us the effective propagation constant K in terms of other known parameters, and this relationship is known as the dispersion relation given by

$$K = \frac{1}{d}\cos^{-1}\left[\begin{array}{l} \cos(k_1 a)\cos(k_2 b) \\ -\frac{1}{2}\left(\gamma + \frac{1}{\gamma}\right)\sin(k_1 a)\sin(k_2 b) \end{array}\right] \quad (6)$$

The effective propagation constant K given by equation (6), can be written as a function of ω and β

$$K(\omega,\beta) = \frac{1}{d}\cos^{-1}\left[\begin{array}{l} \cos(k_1(\omega,\beta).a)\cos(k_2(\omega,\beta).b) \\ -\Delta(\omega,\beta)\sin(k_1(\omega,\beta).a)\sin(k_2(\omega,\beta).b) \end{array}\right] \quad (7)$$

where,

$$\Delta(\omega,\beta) = \frac{1}{2}\left[\frac{\{k_1(\omega,\beta)\}}{\{k_2(\omega,\beta)\}} + \frac{\{k_2(\omega,\beta)\}}{\{k_1(\omega,\beta)\}}\right] \quad (8)$$

The reflection and transmission can be related easily between the plane wave amplifications.

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = M \begin{pmatrix} 1 \\ r \end{pmatrix} \quad (9)$$

and $M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$

with $M_{11} = m_{11}U_{N-1} - U_{N-2}$, $M_{21} = m_{21}U_{N-1}$, $M_{12} = m_{12}U_{N-1} - U_{N-2}$, $M_{22} = m_{22}U_{N-1} - U_{N-2}$

and $U_N = \dfrac{\sin[(N+1).K(\omega,\beta).d]}{\sin[K(\omega,\beta).d]}$

Here N is the number of unit cells of the structure. The transmission and reflection coefficients are given by

$$t = M_{11} - \frac{M_{12}.M_{21}}{M_{22}} \text{ and } r = \frac{M_{21}}{M_{22}} \quad (10)$$

and the associated transmittance (T) and reflectance (R) are obtained by taking the square of the absolute value of *t* and *r* respectively, given by

$$T = |t|^2 \text{ and } R = |r|^2 \quad (11)$$

In a photonic crystal only group velocity (V_g) has a proper meaning and it governs the energy flow of a light beam. The Bloch wave vector in photonic crystal is given by $k = \beta.\hat{x} + K.\hat{z}$. So the group velocity (V_g) in photonic crystal is expressed as [21]

$$V_g = V_{gx}.\hat{x} + V_{gz}.\hat{z} \quad (12)$$

where V_{gx} and V_{gz} are the components of the group velocity in the x- and z-axis respectively and these components are given by

$$V_{gx} = \frac{\partial\omega}{\partial\beta} = -\frac{\partial K(\omega,\beta)/\partial\beta|_{constt.(\omega)}}{\partial K(\omega,\beta)/\partial\omega|_{constt.(\beta)}}$$

$$\text{and } V_{gz} = \left(\frac{dK}{d\omega}\right)^{-1} \quad (13)$$

Therefore the resultant group velocity is given by

$$V_g = \sqrt{(V_{gx})^2 + (V_{gz})^2} \quad (14)$$

and the phase velocity is given by the relation

$$V_p = \frac{\omega}{K(\omega,\beta)} \tag{15}$$

In the next section, we plot dispersion relation, transmittance, group velocity and phase velocity versus normalized frequency using Equations (7), (11), (13) and (15) respectively only for the TE polarization.

III. RESULTS AND DISCUSSIONS

In this section, it will be shown from the results of computations that it is possible to achieve negative refraction in one dimensional photonic band gap material without meta-materials (materials having simultaneously negative permittivity and permeability). For the numerical calculations, we have taken Na_3AlF_6 as dielectric with Ge. The refractive indices of Na_3AlF_6 and Ge are n_1 = 1.34 and n_2 = 4.2 and their thicknesses are a = 0.75% of d and b = 0.25% of d respectively, where d is the total stack thickness. The total number of unit cells N is taken to be equal to 10.

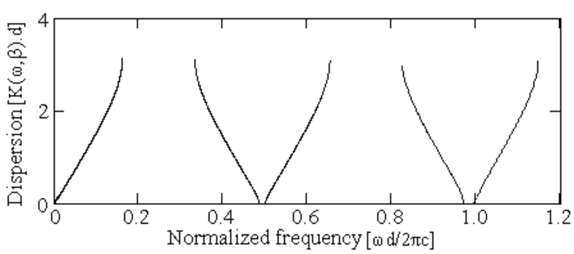

Figure 2: Plot of the dispersion relation as a function of normalized frequency for Na_3AlF_6/Ge at incident angle θ_0=20°.

Figure 2 shows photonic band structure versus normalized frequency for an oblique angle of incidence θ_0=20°. It is clear from Figure 2 that the bandwidths of odd numbered forbidden band gaps are wide but the bandwidths of even numbered forbidden band gaps are extremely narrow. It is noticeable here that the lattice constant d is arbitrary; thus the result obtained here is valid for arbitrary wavelengths and the existence of bandgap is possible as long as $d \approx \lambda$.

Figure 3 shows the plot of the x-component of group velocity V_{gx} versus normalized frequency.

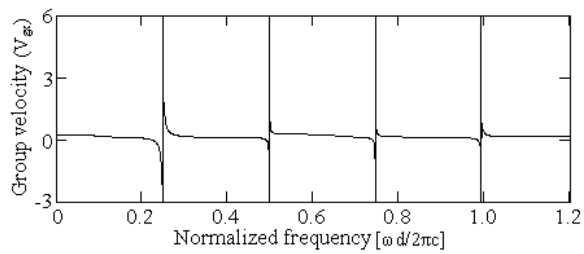

Figure 3: Variation of the x-component of the group velocity as a function of normalized frequency for Na_3AlF_6/Ge at incident angle θ_0=20°.

On comparing Figures 2 and 3, it is clear that there is strong group velocity dispersion at the bandgap edges. With the increase of frequency, V_{gx} decreases from a positive to negative value and falls sharply to the negative minimum at the edge of band and then jumps to the positive maximum. Afterwards V_{gx} decreases again and cycles in this manner. Here, $V_{gx} < 0$ indicates that the energy flow may tend to the negative direction of x-axis, so negative refraction phenomenon may occur at some frequency in the bandgap. At normal incidence V_{gx} is always equal zero, hence the oblique incidence of the wave is the necessary condition for negative refraction.

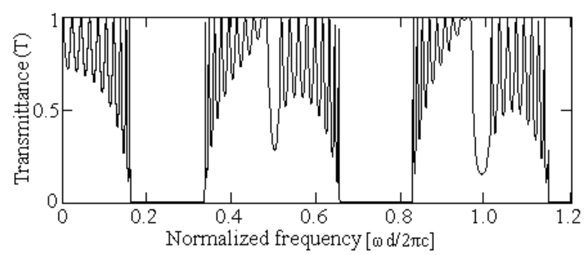

Figure 4: Tranmittance vs normalized frequency for Na_3AlF_6/Ge at incident angle θ_0=20°.

From Figure 4, we observe that the transmittance is zero in the first and third bandgap, i.e. at these frequencies, the waves are completely reflected and do not pass through the crystal but in the second and fourth band gaps, the transmittance is not zero, which means part of the wave can pass. Relating with Figure 3, we observe that there exists a negative group velocity in the low frequency edge of the second bandgap, at the same time; the transmittance is not zero, so the transmitted wave will bend to the negative direction of the x-axis, which is similar to the phenomenon of negative refraction phenomenon. According to the parameters in

Figure 3, the normalized frequency band for negative refraction lies between 0.484ωd/2πc and 0.5ωd/2πc in the second band gap and 0.974ωd/2πc and 0.996ωd/2πc in the fourth band gap, but the transmission energy is less than 5%. However, in the first and third band gaps, the group velocity is negative for certain range, so there is no transmission of energy and the negative refraction can not occur. Therefore, we can conclude that negative refraction may occur in the frequency near the low frequency edge of the second and fourth band gap because of the strong group velocity dispersion.

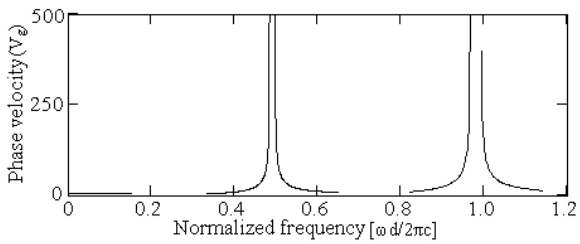

Figure 5: Phase velocity vs normalized frequency for Na_3AlF_6/Ge at incident angle θ_0=20°.

Figure 5 shows the phase velocity versus normalized frequency. The phase velocity in the second and fourth band gaps is infinite because in these regions, the wave is evanescent, the wave number is complex and only the imaginary part exists. Whereas in theory, the evanescent wave decays to zero in infinite photonic crystal, so the phase of the wave does not exist therefore phase velocity has no meaning. In the first and third band gaps, the wave number is complex, but the real part is not zero, so the phase velocity exists, and the phase of the wave varies because of the reflection at interface boundary, the waves are all reflected and can not pass through photonic crystal.

IV. CONCLUSION

It can be concluded that it is possible to achieve negative refraction in one dimensional photonic band gap material without meta-materials (materials having simultaneously negative permittivity and permeability). We demonstrated theoretically that negative refraction may occur near the low frequency edge of the second and fourth band gaps in a 1D photonic crystal having Na_3AlF_6/Ge as alternate layers, for an oblique incidence of electromagnetic wave. This is remarkable in the sense that we need not use exotic negative materials to observe negative refraction all the time. Ordinary materials like metal and other dielectric materials may be used to observe negative refraction. These unique properties of refracting Bloch photons may be exploited in the design of novel photonic devices which may have applications in integrated photonic systems.

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