A Novel Planar 2-D Unit Structure of Double Negative Material

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Abstract — A novel unit of planar distributed structure with double negative materials (DNM) is proposed. The new unit consisted of microstrip line is of composite structure only on single-layer but not two layers as did in the literature. The new structure greatly debases the left-handed frequency by using inter-digital capacitor instead of the old mushroom capacitor as well as with conductive coil as an inductance to the ground rather than the old conductive cylinder. In addition, a new method to extract parameters of the structure is presented. And the equivalent circuit parameters have been extracted from full-wave simulation and composite right/left-handed transmission line (CRLH-TL) theory.

Index Terms—Left-handed material (LHM), composite right/left-handed transmission line (CRLH-TL), Double negative material (DNM)

I. INTRODUCTION

The left-handed material (LHM) is a material whose permeability and permittivity are simultaneously negative [1], therefore it is also called double negative material (DNM). Though LHM has not been found in the nature so far, recently, researches demonstrated the practical realization of LHM with left-handed transmission lines (LH-TLs) using non-resonant, low-loss and broad-bandwidth structure, which have made it possible to apply LH-TL in MIC and MMIC design [2-3]. Because of unavoidable parasitic series inductance and shunt capacitance resulting in a right-handed properties’ contribution with the frequency increasing, therefore the LH-TL is actually a CRLH-TL structure.

The planar distributed 2-D DNM structures have been presented in [4] using one or two layer structures. However, the previous one layer structure can only obtain higher LH frequency, whereas the two layer structure is rather complicated though it can lower the LH frequency. In this paper, a novel unit planar structure with single-layer is proposed. The present unit structure exhibits strongly enhanced left-handedness due to inter-digital capacitor and coil inductance rather than the single-layer structure given in [4]. Meanwhile, a new method to extract parameters of the structure is presented, and the equivalent circuit parameters of the unit structure have been extracted by means of full-wave simulation.

II. UNIT STRUCTURE

The proposed planar unit structure of double negative material is shown in Fig.1. The unit cell consists of a top square inter-digital capacitors patch with a coil to the ground plane at the center. The top patch provides capacitive couplings with the adjacent patches and the coil provides an inductance, which yields the LH
characteristics.

The inter-digital capacitors greatly enhance the capacitance to the adjacent cells and, consequently, the LH nature is enhanced. In addition, the coils connecting the microstrip lines to the ground plane act as the shunt inductors whose inductance is much higher than the old one, which further enhance the LH nature. Consequently, the operation frequency is greatly pushed down, which is difficult to achieve with a single-layer structure in the former structure. The present structure becomes isotropic in two dimensions when the guided wavelength is large compared with the size of unit cell. On the other hand, one can intentionally make the structure anisotropic by changing the inter-digital capacitor shape to form different capacitance values in different directions. Although the mode guided by the structure is microscopically hybrid, it is essentially compatible with the TEM mode and can be excited easily by a microstrip line or PPW.

III. THEORY ANALYSIS

A. Dispersion Relation

The equivalent circuit of the unit cell of the proposed 2-D structure is shown in Fig.2.

Expressing the circuit by a four-port ABCD-matrix and applying the Bloch-Floquet periodic boundary condition, we have the dispersion relation by solving the eigenvalue

Fig.2. Equivalent circuit of the unit cell of the 2-D CRLH-TL

Expressing the circuit by a four-port ABCD-matrix and applying the Bloch-Floquet periodic boundary condition, we have the dispersion relation by solving the eigenvalue
problem [5] as
\[
\frac{(e^{jka} - 1)^2}{e^{-jka}} + \frac{(e^{jka} - 1)^2}{e^{-jka}} - 2Z(\omega)Y(\omega) = 0
\]  
(1)
and
\[
\beta = \sqrt{k_x^2 + k_y^2}
\]  
(2)
where \(k_x\) and \(k_y\) are the wave-numbers in the x- and y-directions and \(Z(\omega)\) and \(Y(\omega)\) are the series impedance and shunt admittance of the branches. In the particular case of the CRLH unit cell of Fig. 1, the immittances are given by
\[
Z(\omega) = \frac{1}{2j \omega C_L} + j \omega R \quad Y(\omega) = \frac{1}{2j \omega L} + j \omega C_R
\]  
(3)
The wave-numbers \(k_x\) and \(k_y\) along the Brillouin zone [7] are obtained from (1) as
\[
k_x = \frac{1}{a} \cos^{-1}\left\{1 - \frac{1}{2} \left[\frac{\omega_x^2}{\omega_L^2} + \frac{\omega_x^2}{\omega_R^2} - p\right]\right\}
\]  
\(\Gamma - X : 0 < k_x a < \pi; \ k_x a = 0\)
(4)
\[
k_y = \frac{1}{a} \cos^{-1}\left\{3 - \frac{1}{2} \left[\frac{\omega_y^2}{\omega_L^2} + \frac{\omega_y^2}{\omega_R^2} - p\right]\right\}
\]  
\(X - M : 0 < k_y a < \pi; \ k_y a = \pi\)
(5)
\[
k_i = \frac{1}{a} \cos^{-1}\left\{1 - \frac{1}{4} \left[\frac{\omega_i^2}{\omega_L^2} + \frac{\omega_i^2}{\omega_R^2} - p\right]\right\}
\]  
\(M - \Gamma : 0 < k_i a = k_y a < \pi; \ i = x \ and \ y\)
(6)
where
\[
\omega_L = \frac{1}{\sqrt{L_{L}C_L}} \quad \omega_R = \frac{1}{\sqrt{L_{R}C_R}}
\]  
(7)
The parameters \(L_p\) and \(C_p\) are discussed in part B.

The theoretical dispersion diagram is shown in Fig. 3. We can see that it supports a fundamental backward wave (LH mode) in the frequency range from \(f_{f1}\) to \(f_{M1}\), in which it exhibits anti-parallel phase velocity \(\omega / \beta\) and group velocity \(\partial \omega / \partial \beta\). On the other hand, in the frequency range from \(f_{f2}\) to \(f_{M2}\), we have the forward wave (RH mode with parallel group and phase velocities). Below \(f_{M1}\) and above \(f_{M2}\) are the stop-bands due to the high pass of LH and the low pass of the RH natures of the CRLH unit cell, respectively [4].
We will see in the simulation afterwards that the frequency at $\Gamma$ point can not be obtained accurately due to the effect of the air mode. Therefore it is needed to find a new approach to extract parameters, which is different from the way used in [10]. The parameters $\omega_L$, $\omega_R$, and $p$ can be extracted from the angular frequency $\omega_{N_1}$, $\omega_{X_1}$, and $\omega_{M_1}$. The three angular frequencies satisfy the dispersion relation (4) (5):

\[
\frac{\omega_L^2}{\omega_{N_1}^2} + \frac{\omega_L^2}{\omega_{X_1}^2} - p = 2
\]

\[
\frac{\omega_L^2}{\omega_{N_1}^2} + \frac{\omega_L^2}{\omega_{X_1}^2} - p = 4
\]

\[
\frac{\omega_L^2}{\omega_{N_1}^2} + \frac{\omega_L^2}{\omega_{X_1}^2} - p = 8
\]

from (8) (9) (10), we can obtain

\[
\omega_L = \sqrt{\frac{\Delta_L}{\Delta}}
\]

\[
\omega_R = \sqrt{\frac{\Delta_R}{\Delta}}
\]

\[
p = \frac{\omega_L^2}{\omega_{X_1}^2} + \frac{\omega_L^2}{\omega_{X_1}^2} - 4
\]

where

\[
\Delta = \begin{vmatrix}
\frac{1}{\omega_{X_1}^2} & -\frac{1}{\omega_{M_1}^2} & \omega_{X_1}^2 - \omega_{M_1}^2 \\
\frac{1}{\omega_{X_1}^2} & -\frac{1}{\omega_{M_1}^2} & \omega_{X_1}^2 - \omega_{M_1}^2 \\
\frac{1}{\omega_{X_1}^2} & -\frac{1}{\omega_{M_1}^2} & \omega_{X_1}^2 - \omega_{M_1}^2 \\
\end{vmatrix}
\]

\[
\Delta_L = \begin{vmatrix}
-4 & \omega_{X_1}^2 - \omega_{M_1}^2 \\
6 & \omega_{M_1}^2 - \omega_{N_1}^2 \\
\end{vmatrix}
\]

B. parameter $p_L$ and $p_C$

First we define that $\omega_R = 1$, which means that the following frequencies and the bandwidth are normalized with $\omega_R$:

The relationships of central LH frequency $\omega_b$, LH bandwidth $\Delta \omega$, relative bandwidth $\Delta \omega / \omega_b$ and the parameters of $p_L$ and $p_C$ are shown from Fig.4 to Fig.6.
Fig. 5 Relation between LH bandwidth $\Delta \omega$ and $p_L$ and $p_C$

Hence, there are two methods to reduce the central LH frequency, one is to decrease $p_c$, the other is to decrease $p_L$. Obviously, the prior one is more effective.

However, if $p_c$ is reduced, the bandwidth $\Delta \omega$ will also decrease (Fig. 5), and so will the relative bandwidth $\Delta \omega / \omega_0$ (Fig. 6). It is shown in Fig. 5 that $\Delta \omega$ increases with the decrease of $p_L$, especially when $0 \leq p_L \leq 1$. Consequently, both $p_L$ and $p_C$ account for the pattern of relative bandwidth $\Delta \omega / \omega_0$. From Fig. 6 we can see that $\Delta \omega / \omega_0$ is mostly controlled by $p_L$; it increases with the decrease of $p_L$.

Therefore, $p_C$ controls the central LH frequency $\omega_0$ while $p_L$ determines the relative bandwidth $\Delta \omega / \omega_0$. To obtain a lower central operated frequency and a higher relative bandwidth, both $p_L$ and $p_C$ should be decreased. Compared with the structure given in [4] the contemporary structure is optimized in both $p_L$ and $p_C$. As a result the present structure has a much lower central frequency and a much higher relative bandwidth which will demonstrate in the next part.

IV. SIMULATION RESULTS

The proposed periodic structures shown in Fig. 1 are designed. The period of the unit cell is $5 \times 5 \text{mm}^2$, which is the same size with the former structure [4], and the dimensions of the interdigital capacitors are shown in the inset of the figure. The relative permittivity of the substrate is chosen as 2.2.

The dispersion diagram is calculated for the unit cell shown in Fig. 2 by full-wave FEM simulations. The simulated dispersion diagram
for the presented structure is shown in Fig. 7, a pure LH mode is observed as the fundamental mode in the circuit model. However, when $\beta$ is close to $k_0(=\omega/c_0)$, the LH mode couples with the air mode so that the dispersion curve is bended down in the coupling region. The higher mode appearing just above the LH mode is a degenerated TE mode and it is not the RH mode; but the next higher mode is.

![Dispersion diagram of the unit obtained with full-wave simulation](image)

In Fig. 7, the LH operation frequency is about from 1.6GHz ~ 3.6GHz. In the former single-layer structure [4], the LH operation frequency is about from 5GHz ~ 6.5GHz. The LH nature is enhanced and the operation frequency is greatly pushed down for the present single-layer structure. The equivalent circuit parameters of the proposed unit cell are calculated according to (11) (12) (13), $\omega_0 / 2\pi = 5.3051\text{GHz}$, $\omega_{\text{rh}} / 2\pi = 4.5594\text{GHz}$ and $p= 2.3219$. The dispersion diagram calculated are shown in Fig.8, it can be seen that the dispersion characteristics calculated with proposed theory for the LH mode agree very well with the full-wave simulation results.

![Dispersion diagrams obtained the full-wave simulation and 2-D CRLH-TL theory](image)

The practical central LH frequency $\omega_0$ of the proposed structure is about 2.6GHz and the relative bandwidth $\Delta\omega / \omega_0$ is about 77%. The LH frequency of the structure given in [4] is about 5.75GHz and the relative bandwidth is about 26%. Just agreeing with the proposed theory, the present structure has a lower central LH frequency and a higher relative bandwidth.

V. CONCLUSION

A novel unit of planar structure with double negative material is proposed, which is based on the microstrip and metallic coil. By using inter-digital capacitor and coil inductance instead of the former mushroom structure, the novel unit has not only greatly debased the LH transmission frequency but also enhanced the relative bandwidth compare with the former structure. The dispersion diagram has been simulated by full-wave simulations and composite transmission line theories, also the equivalent circuit parameters have been
REFERENCES


