Comparison of Optimization Techniques for Square Split Ring Resonator

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Abstract- In this paper, three optimization techniques namely genetic algorithm (GA), Artificial Neural networks (ANN) and hybrid GA-ANN is applied to the case of square split ring resonator. The obtained results are compared. In all the cases, the size of the split ring resonator is optimized in order to obtain minimum error in resonant frequency. There are many types of GA and ANN. Here, Binary genetic algorithm and feed forward backpropagation ANN is chosen. A new method of hybridization is presented here which gives effective results.

Index Terms- Artificial neural network (ANN), Binary genetic algorithm (BGA), Equivalent Circuit Analysis (ECA), Hybrid GA-ANN and Square split ring resonator (SSRR).

I. INTRODUCTION

Metamaterials have been one of the popular areas of research in the field of microwaves in the recent past. The reason is that they exhibit many unusual properties like negative refractive index, reversed Snell’s law and Doppler Effect etc. By definition, metamaterials are artificially engineered materials which satisfying the homogeneity limit, exhibiting properties not usually found in nature [1]. The first proposition of this was given by the Russian physicist Prof.V.Veselago in 1968 [2]. In those days, practically realizing such materials was not possible. The real breakthrough came when J.B.Pendry et.al. Around the year 2000 [3] - [4] showed that an array of concentric circular rings and wire strip could exhibit negative permeability and permittivity and hence negative refractive index. From there on, various structures of split ring resonators were brought to the fore like square, omega shaped, U-shaped resonators etc.

Optimization is one of the important techniques used before designing. We present optimization of the square split ring resonators by three techniques. The size of the resonator that is the side of the square, the conductor width and the dielectric spacing are the parameters which are optimized in order to achieve the proper resonant frequency.

This paper henceforth is divided into six sections. The second section gives the structure of the resonator; the third describes the equivalent circuit analysis and the theory behind the square split ring resonator; the fourth describes the optimization using GA, ANN and the theory behind hybridization of the two above mentioned techniques. The fifth and sixth sections describe the results obtained and finally the conclusion.

II. STRUCTURE OF THE RESONATOR

The structure of the square split double ring resonator is as shown below along with the dimensions as given in previous literature [5].

![Fig.1. Structure of Square split ring resonator](image)

It consists of two square shaped rings with gaps in between them on the opposite side of both the rings. This structure is printed on a substrate...
of dielectric thickness of 1.5625mm and dielectric constant of 2.3 (RT Duroid). Here, ‘a’ denotes the length of the side of the square, ‘w’ denotes the width of the conductor, ‘d’ denotes the dielectric width between the inner and the outer square and ‘g’ denotes the gap present in the rings. Care is taken that the gap width ‘g’ does not change from the inner ring to the outer ring.

III. EQUIVALENT CIRCUIT ANALYSIS

One method of obtaining the resonant frequency is by using the equivalent circuit analysis (ECA) method as prescribed in [6]-[7]. The given distributed network is converted into lumped network and analyzed.

When a magnetic field is applied perpendicular to the plane of the ring, the ring begins to conduct and gives rise to current flow. The current flowing through the rings will enable it to act as an inductor and the dielectric gap (d) between the rings will lead to mutual capacitance. Hence the equivalent circuit of the SSRR will be a parallel LC tank circuit as shown in Fig.2.

![Fig.2. Lumped equivalent circuit of SSRR](image)

From this, we can calculate the resonant frequency.

\[ f_0 = \frac{1}{2\pi \sqrt{LC_s}} \]  (1)

Where \( C_s \) is the equivalent capacitance and \( L \) is the effective inductance due to both of the rings.

The expressions for effective inductance and capacitances can be obtained from [8] as follows:

\[-\frac{8.46 \mu_0}{2} \left( a - w - d \right) \left[ \ln \left( \frac{0.98}{\rho} \right) + 1.84 \rho \right] \]  (2)

Where, \( a, w, d \) are the notations prescribed in the previous section, \( \rho \) is the filling factor of the inductance and is given by

\[ \rho = \frac{w + d}{a - w - d} \]  (3)

The effective capacitance is given by

\[ C_s = \left( a - \frac{3}{2} (w + d) \right) C_{\text{pul}} \]  (4)

Where, \( C_{\text{pul}} \) is the per-unit-length capacitance between the rings which is given as below

\[ C_{\text{pul}} = \varepsilon_0 \varepsilon_{\text{eff}} \frac{K(1 - k^2)}{K(k)} \]  (5)

Here, \( \varepsilon_{\text{eff}} \) is the effective dielectric constant which is expressed as

\[ \varepsilon_{\text{eff}} = \frac{\varepsilon_r + 1}{2} \]  (6)

\( K(k) \) denotes the complete elliptical integral of the first kind with \( k \) expressed as

\[ k = \frac{d}{d + 2w} \]  (7)

Using the above expressions, the lumped equivalent of the SSRR is modeled. We are also able to infer that the circuit elements and hence the resonant frequency, depend on the dimensions of the resonator.

IV. DESCRIPTION OF OPTIMIZATION TECHNIQUES

A brief theory of all the three types of optimization techniques and how it is implemented for SSRR is given below.

A. Binary Genetic Algorithm (BGA)

The Genetic algorithm is an optimization technique based on evolution. It was first invented by John Holland [9]. It is one of the means of producing a global optimal solution with inspiration drawn from nature. Many variations are now-a-days present in Genetic
algorithm but the original being the binary genetic algorithm [10]. Some of the applications of genetic algorithms for optimization problems in microwaves are listed in [11]-[12].

It encodes all the parameters which are to be optimized into binary strings. Each binary string represents a chromosome. Likewise, many such chromosomes are collected and a pool of possible solutions is obtained. This set of solutions is called as population. For each chromosome in this population, the fitness index is calculated. Depending on the fitness index, the best solution is selected. Two main operations which are performed are the cross over and the mutation process which enables to find the best solution with great accuracy. The cross over operation helps to obtain best possible solutions from the given population and mutation process helps to explore for new solutions and prevents the solution from settling in the local optimum.

The main advantage of using genetic algorithm is that it is a non-gradient type of optimization and hence will not search a narrow solution set. Hence the global optimal solution could be obtained. Also, it does not allow the solution to settle into local optima.

A short procedure on the working of BGA is as given below.
Step1: Create an initial population of possible solutions in a binary code form (chromosomes).
Step2: Calculate the actual decimal value, from it the fitness index of each chromosome present in the generation.
Step3: Rank the chromosomes in the order of the fitness. For minimization problem, the order will be ascending order and for maximization, descending order is chosen.
Step4: While we can still search for optimal value or the number of generations is less than the maximum number of generations specified, go to step 5; else go to step 8.
Step5: Select a set of the best fit population for crossover depending upon the probability of crossover.
Step6: Select the random positions in the random chromosomes in the population and toggle the bits. This is the mutation process.
Step7: Go to step 2.
Step8: Extract the optimal values from the chromosomes.

In this paper, binary genetic algorithm is used in a steady state style. That is, a portion of the population is carried over in each generation. But this helps in quicker convergence.

B. Artificial neural networks (ANN)

The artificial neural network is an information processing system. The network is made up small computational elements called as neurons. It has an input and transfer function leading to the output. Many neurons are thus interconnected by means of synapse or weights. When input is presented to the ANN, it tries to adjust the weights in order to match the target vector. There are many structures of neural network and may also have many layers. Mostly for microwave applications, three or four hidden layer structures are used along with one input and one output layer [13]-[14]. The structure of the neural network is as shown in the diagram below.

![Fig.3. Basic structure of ANN](image)

The main advantage of the neural network is that it produces near optimal solution quicker and has many variations which can be tried on even though it is not a better optimizer. Also, given a small search space, it will produce the best optimal solution quickly as most of the structures are of a gradient type.

For the determination of resonant frequency of a given SSRR, a feed-forward backpropagation network is used. A three hidden layer network is used in accordance to the universal approximation theorem that a three hidden layer network can approximate any parameter to any accuracy.

C. Hybridization of GA and ANN

The reasons behind hybridization of GA with ANN are as given in brief below. GA provides better search space and accuracy whereas neural
network provide quick solutions. Therefore by combining the two techniques, the advantages of both could be utilized and an effective structure could be produced. There are various methods of hybridization could be adopted.

Examples of hybridization of GA-ANN for microwave applications are as given in [15]-[16]. For example, determining the optimal weights of the artificial neural network using GA to enable faster training of the ANN. While this may be able to work in an efficient manner with moderately complex problems, it would have a great drawback when it comes to modeling a complex non linear system as we would have few data and proper equations prescribing it will be absent.

A new method of hybridization is proposed here wherein ANN is used for obtaining the fitness of a particular chromosome. This would help in GA performing better for complex systems. While calculating the fitness of a chromosome in the absence equations, we can provide the parameters of the chromosome to a trained artificial neural network for generating the output and fitness function. The above approach would act as a good method of hybridization while modeling complex systems with few data sets.

For modeling the SSRR by hybridization of GA-ANN, we first train a back propagation network which accepts the input parameters as the dimensions of the resonator as input (which is the same parameters that are encoded in GA) and give the output as the fitness function and the output. Now, when we want to calculate fitness for each chromosome, we refer to the network. Thus the problem is approached and the results are presented in the paper. We are able to infer that the results presented as expected, able to extract the advantages of both the GA and the ANN methods.

V. RESULTS OBTAINED AND INFERENCES

Firstly, the resonant frequency is calculated for various dimensions with $\varepsilon_r = 3.86$ according to the equivalent circuit model presented in section III. The inductor, capacitor and the resonant frequency will vary according to the dimension of the SSRR. This forms the data set for training the ANN and obtaining the optimized values using GA. The results thus obtained with equivalent circuit analysis (ECA) are compared with all the three optimization techniques. We shall see the results of each technique.

A. Results obtained through Binary Genetic Algorithm (BGA)

The results obtained by using BGA for optimization of the size of the resonator namely the side length ($a$), width of the conductor ($w$) and the spacing between the two rings ($d$). The search intervals for all the above parameters are $3 \leq a \leq 16$ mm, $0.1 \leq w \leq 0.9$ mm and $0.1 \leq d \leq 0.9$ mm.

Each of the parameters is represented using ten bits and hence each chromosome would be of 30 bit length. The total population of chromosomes is chosen 300. Steady state style of GA is used meaning a portion of the population is retained every generation and the rest of the population is allowed to participate in the cross over. The crossover probability is chosen as 45% and mutation probability is chosen as 0.2%. The fitness function that was used for this optimization problem is the absolute error in the resonant frequency which is expressed as

$$f_{err} = \frac{|f_d^d - f_0^d|}{f_0^d} \quad (8)$$

Where $f_d^d$ is the desired resonant frequency in GHz and $f_0^d$ is the resonant frequency calculated in GHz.

The obtained parameters of the resonator are as shown below in Fig.4 and Fig.5.

![Fig.4.Optimized values of conductor width and dielectric spacing of SSRR with respect to resonant frequency.](image)
The input buffer layer has three neurons representing the three inputs given to the network namely the side length, conductor width and dielectric spacing of the resonator in mm.

The output layer consists of one neuron for the resonant frequency in GHz. The hidden layers have the transfer function of log-sigmoid function and for output layer it is linear.

The range of inputs for each of the dimension of the Square split ring resonator is $3 \leq a \leq 16\text{mm}$, $0.1 \leq w \leq 0.99\text{mm}$ and $0.1 \leq d \leq 0.99\text{mm}$. This structure is properly trained till the maximum of 3000 epochs is reached. The obtained results for the dimensions are presented below.

**B. Results obtained through Artificial Neural network (ANN)**

The artificial neural model developed for SSRR is a four layer network with the three hidden layers, one input buffer layer and one output layer. The hidden layers consist of five numbers of neurons each. The training algorithm used is Levenberg-Marquardt (LM) algorithm. The values obtained through ANN are as given below.
The variation of the dimensions of the resonator namely the side length, width of the dielectric spacing and the conductor width with the resonant frequency is plotted in Fig.7, Fig.8 and Fig.9 by both the ECA and ANN methods. We are able to infer that there is not much difference between the values computed by ECA and those obtained from ANN on comparison.

C. Results obtained through hybridization of GA-ANN

The parameters are chosen as factors common to both GA and ANN. The network used for generating the outputs and the fitness function is a five layer feed forward back propagation network with proper convergence. The ANN thus trained is used for calculating the fitness index of the data set present in the particular chromosome specified in the population set described by BGA. The inputs represented as chromosomes are the dimensions of the SSRR namely the side length, conductor width and the spacing width. A set of these parameters constitute a chromosome. The chromosomes are chosen to be ten bit length for each parameter and the population of 300 chromosomes is chosen. The crossover probability is 45% (steady state style) and mutation probability is kept as 0.2%. The GA is then allowed to run for ten generations for convergence of the results and for preventing premature convergence on a non-optimal value.

The intervals of inputs for the dimensions of the resonator are $3 \leq a \leq 16 \text{mm}$, $0.1 \leq w \leq 0.99 \text{mm}$ and $0.1 \leq d \leq 0.99 \text{mm}$. The results obtained for the frequency range of 1 GHz to 6 GHz are tabulated as shown.

### TABLE. I
Dimensions of the SSRR obtained as a result of hybridization.

<table>
<thead>
<tr>
<th>$f_o$ (GHz)</th>
<th>a (mm)</th>
<th>W (mm)</th>
<th>d (mm)</th>
<th>$f_o^{obtained}$ (GHz)</th>
<th>$f_{err}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>6.2913</td>
<td>0.6323</td>
<td>0.6894</td>
<td>0.9995</td>
<td>0.0008</td>
</tr>
<tr>
<td>1.5</td>
<td>4.7410</td>
<td>0.3041</td>
<td>0.7167</td>
<td>1.4988</td>
<td>0.0008</td>
</tr>
<tr>
<td>2.0</td>
<td>4.4614</td>
<td>0.4114</td>
<td>0.9604</td>
<td>2.0006</td>
<td>0.0003</td>
</tr>
<tr>
<td>3.0</td>
<td>3.4066</td>
<td>0.4211</td>
<td>0.8575</td>
<td>3.0015</td>
<td>0.0005</td>
</tr>
<tr>
<td>4.0</td>
<td>3.2414</td>
<td>0.6182</td>
<td>0.9525</td>
<td>4.0176</td>
<td>0.0044</td>
</tr>
<tr>
<td>5.0</td>
<td>3.1144</td>
<td>0.7589</td>
<td>0.9375</td>
<td>5.0023</td>
<td>0.0005</td>
</tr>
<tr>
<td>6.0</td>
<td>3.1652</td>
<td>0.9085</td>
<td>0.9648</td>
<td>6.0047</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

The error is also minimal only as compared to both the above techniques being used separately which are evident from the above table.

Taking into account the time taken for execution of the program, accuracy of the obtained result and the amount of memory required for its functioning, the three optimization techniques are compared as shown below. The accuracy is depicted below in terms of the performance characteristics obtained for the entire set of data. The simulations and the comparisons were done using Matlab 7.7.0.

### TABLE. II
Comparison between the optimization techniques presented.

<table>
<thead>
<tr>
<th>Optimization Technique</th>
<th>Accuracy</th>
<th>Execution Time (Sec)</th>
<th>Memory Needs (MB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BGA</td>
<td>0.007</td>
<td>149.549</td>
<td>44</td>
</tr>
<tr>
<td>ANN</td>
<td>0.0241</td>
<td>120.446</td>
<td>40</td>
</tr>
<tr>
<td>Hybrid GA-ANN</td>
<td>0.0044</td>
<td>83.1105</td>
<td>48</td>
</tr>
</tbody>
</table>

From the above table, we understand the trade-off between the criteria of accuracy, execution time and memory requirement. Depending on the need, we are able to select the technique needed for the problem.
VI. CONCLUSION

The three optimization techniques of binary genetic algorithm, artificial neural network and hybridization of genetic algorithm and ANN were applied to a square split ring resonator. The data set is generated from the ECA analysis of the SSRR and the results obtained for each of the technique was compared with the data set obtained through ECA. The error is also well within the tolerable limits. In all cases, the parameters that are optimized are the three dimensions of the SSRR namely, the side length, conductor width and dielectric spacing. Optimization is done with respect to a desired resonant frequency of the resonator. A new method of GA-ANN hybridization was studied and was found effective for modeling complex systems.

REFERENCES