Behavior of Reflected Powers of Radiation Field in Shorter Wavelength Region in Various Kinds of 3-D Optical Waveguides with Periodic Structure

Tokuo Miyamoto†*, Michiko Momoda†, and Kiyotoshi Yasumoto††

†Department of Electronics Engineering and Computer Science, Faculty of Engineering, Fukuoka University, 8-19-1 Nanakuma, Jonan-ku, Fukuoka-shi, 814-0180 Japan
††Faculty on Information Science and Electrical Engineering, Kyushu University, 6-10-1 Hakozaki, Higashi-ku, Fukuoka-shi, 812-8581 Japan
E-mail: tmiyamot@fukuoka-u.ac.jp, mm@fukuoka-u.ac.jp and yasumoto@csce.kyushu-u.ac.jp

Abstract- In the investigation of wavelength characteristics on reflected power of dominant guided mode in various kinds of 3-D periodic waveguides, the increase of reflected radiation field in the shorter wavelength region has been one of problems, and some suppression techniques have been considered. In this paper, behaviors of such a radiation field in 3-D waveguides with periodic structure which are index-modulation type and lamellar grating type with rectangular teeth and with sinusoidal surface relief are investigated using Fourier series expansion method. According to the numerical results, their characteristics behaviors for each periodic waveguide are made clear, and results for suppression of the radiation fields are also exemplified.

Index Terms- numerical analysis, 3-D periodic waveguides, reflected radiation field.

I. INTRODUCTION

Various kinds of three dimensional optical waveguides with periodic structure have been playing an important role in optical IC, and the analysis with higher accuracy on such waveguides with arbitrary number of periods has been required. However their accurate analysis with larger number of periods is not always easy, and is seemed to be few. On the other hand, Fourier series expansion method (FSEM) which is effective for full-vectorial analysis not only on transmitted and guided waves but also on reflected and radiation waves has been applied to the numerical analysis on various kinds of three dimensional waveguide systems [1], including periodic optical waveguides with rectangular cross-section[2-5]. In such investigations of the wavelength characteristics on reflected powers of dominant guided mode and discretized radiation modes in each periodic waveguide, the increase of radiation field in the shorter wavelength region has been one of problems.

In this paper, using FSEM method, we tried detailed investigations paying attention to the wavelength characteristics in the shorter wavelength region on reflected radiation power in several kinds of three dimensional optical waveguides with periodic structure which are index-modulation type and lamellar grating type with rectangular teeth and with sinusoidal surface relief. Then the characteristic behavior on the radiation field in shorter wavelength region is made clear for each periodic waveguide, and the methods for suppression of such a radiation field are also exemplified.

II. FOURIER SERIES EXPANSION METHOD

For convenience, we consider the following normalized Maxwell equations [1],[2]

\[ \nabla \times \mathbf{E}(x,y,z) = -j \mathbf{H}(x,y,z), \]
\[ \nabla \times \mathbf{H}(x,y,z) = j \varepsilon(x,y) \mathbf{E}(x,y,z) \] (1)

where \( \varepsilon(x,y) \) is the distribution of relative permittivity of the entire medium in the cross section considered. In order to derive the electromagnetic field by solving (1) using FSEM, virtual periodic boundaries with periods \( \Lambda_x \) and
\( \Lambda_y \) are introduced as shown in Fig.1. To simplify the explanation of the algorithm, we first consider one uniform region (I or II) in the periodic structure as a single waveguide system. Subsequently the algorithm is applied as a problem of periodic structure by taking the connection of each uniform region into consideration.

The electromagnetic field components satisfying (1) can be expressed by the following complex double Fourier polynomials:

\[
E(x, y, z) = \sum_{m=-M}^{M} \sum_{n=-N}^{N} e_{m,n}(z) \exp(-jmx) \exp(jtny),
\]

\[
H(x, y, z) = \sum_{m=-M}^{M} \sum_{n=-N}^{N} h_{m,n}(z) \exp(-jmx) \exp(jtny),
\]

\[
i = x, y, z, \quad s = 2\pi/\Lambda_x, \quad t = 2\pi/\Lambda_y.
\] (2)

Substituting (2) into (1), following matrix form of first order differential equations are derived:

\[
de(z)/dz = -jC_h(z),
\]

\[
dh(z)/dz = -jC_e(z)
\] (3)

where the transverse Fourier coefficients \( \{e_{m,n}(z)\}, \{h_{m,n}(z)\}, (i=x, y) \) are expressed by column vectors \( e(z) \) and \( h(z) \) of order \( 2K(K=(2M+1)(2N+1)) \), respectively. Matrices \( C_x \) and \( C_y \) of order \( 2K \) are composed of diagonal matrices \( [s\delta_{mn} \delta_{mn}] \) and \( [tn\delta_{mn} \delta_{mn}] \), and of cycle matrix \( [e_{m-m,n-n}] \) where \( e_{m-m,n-n} \) is Fourier component of \( e(x, y) \) [1],[2]. To save computational effort, the following second-order differential equation is considered in place of (3):

\[
d^2e(z)/dz^2 = -jC_xC_ye(z).
\] (4)

Equation (4) is solved as an eigenvalue problem of the coefficient matrix \( C = C_xC_y \), then the eigenvalue \( \kappa_i \) and the corresponding eigenvector \( \mathbf{P}^e(k=1,2,...,2K) \) can be obtained readily by a standard subroutine. Here we assume that the waveguide is lossless, and \( \kappa_i^2 \) is a real number. Using the solution \( \mathbf{P}^e \) and the relation \( e(z) = \mathbf{P}^e\mathbf{a}(z) \) where \( \mathbf{P}^e = [\mathbf{P}^{e1}, \mathbf{P}^{e2}, \cdots, \mathbf{P}^{e2K}] \), (4) can be solved, and we obtain the solution on vector form \( \mathbf{f} = [\mathbf{e} \ \mathbf{h}] \) as follows [2]:

\[
f(z) = PU(z - z_0)a(z_0),
\] (5)

\[
P = \begin{bmatrix} \mathbf{P}^e & \mathbf{P}^e \\ \mathbf{P}^h & -\mathbf{P}^h \end{bmatrix}, \quad \mathbf{P}^h = [\kappa_i \delta_{kk}]C_i^+\mathbf{P}^e,
\]

\[
U(\Delta z) = \begin{bmatrix} \exp(-j\kappa_i \Delta z)\delta_{kk} & 0 \\ 0 & \exp(j\kappa_i \Delta z)\delta_{kk} \end{bmatrix},
\]

\[
a(z) = [a^+ (z) \ a^- (z)]', \quad a^+ = [a^+_1 \ a^+_2 \ \cdots \ a^+_2K]'
\]

where \( "^t\) denotes the transpose, \( [\kappa_i \delta_{kk}] \) is the diagonal matrix of order \( 2K \), and \( a^+_k \) expresses the complex amplitude of the \( k \)-th eigenmode corresponding to the normalized propagation constant \( \pm \kappa_i \) in the \( \pm z \) direction.

Next, we consider the connection problem between the solutions \( f^i, a^i, \mathbf{P}^i \) and \( \mathbf{U}^i \) in each uniform region \( i = I, II \) of the periodic structure. The boundary conditions on electro-magnetic fields in the region I and II satisfy \( f(z) = f^i(z) \) within one period \( L \) in Fig.1 [2]. From these relations we derive \( \mathbf{a}^i(L) = \mathbf{F}_a \mathbf{a}^i(0) \), where \( \mathbf{F}_a \) is expressed as

\[
\mathbf{U}^i(\Delta z/2)(\mathbf{P}^i)^{-1} \mathbf{P}^i \mathbf{U}^i(\Delta z)\mathbf{U}^i(\Delta z/2)
\] (6)

and is the transfer matrix of the mode amplitude \( \mathbf{a}^i \) at one period \( L \). Due to the symmetric structure in one period \( L \) of the periodic waveguide as shown in Fig.1, \( \mathbf{F}_a \) is expressed as follows [2]:

\[
\mathbf{F}_a = \mathbf{XV}(L)\mathbf{X}^{-1},
\] (7)

\[
\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 & \mathbf{X}_2 \\ \mathbf{X}_2 & \mathbf{X}_1 \end{bmatrix}, \quad \mathbf{V}(L) = \begin{bmatrix} \mathbf{V}^+(L) & 0 \\ 0 & \mathbf{V}^-(L) \end{bmatrix},
\]

\[
\mathbf{X}_i = [\mathbf{X}_{i,1} \ \mathbf{X}_{i,2} \ \cdots \ \mathbf{X}_{i,2K}], \quad i = 1, 2
\]

where diagonal matrix \( \mathbf{V}^\pm(L) = [\exp(\mp j\gamma_i L)\delta_{ij}] \) represents the eigenvalue of matrix \( \mathbf{F}_x \), and the corresponding eigenvectors are expressed by \( \mathbf{X}_{i,j} \) for eigenvalue \( \exp(-j\gamma_j L) \) and \( \mathbf{X}_{i,j} \) for \( \exp(j\gamma_j L) \) [2].
Referring (7) and the relation \( \mathbf{b}^D(z) = \mathbf{X}^{-1} \mathbf{a}^D(z) \), we obtain amplitude vector of Floquet mode \( \mathbf{b}^d(z) = \mathbf{V}(L)\mathbf{b}^d(0) \). In the case of grating with finite length consisting of \( N_g \) periods, \( \mathbf{b}^d(N_g L) = \mathbf{V}(N_g L)\mathbf{b}^d(0) \) is obtained. This relation can be rearranged in terms \( \mathbf{a}^D(z) \) as

\[
\begin{bmatrix}
1 & 0 \\
\mathbf{V} & \mathbf{0} \\
0 & 1
\end{bmatrix}
\mathbf{x}^{-1}
\begin{bmatrix}
\mathbf{a}^d(N_g L) \\
\mathbf{a}^d(0)
\end{bmatrix}
= \begin{bmatrix}
\mathbf{V}^D(N_g L) & \mathbf{0} \\
\mathbf{0} & 1
\end{bmatrix}
\mathbf{x}^{-1}
\begin{bmatrix}
\mathbf{a}^d(0) \\
\mathbf{a}^d(0)
\end{bmatrix}.
\]

(8)

Here, relation \( \mathbf{b}^D(0) = \mathbf{V}^D(N_g L)\mathbf{b}^D(N_g L) \) is substituted. It is noted that, however large \( N_g \) may be, the value of \( N_g \) has almost no effect on the computing time.

In the case of dominant mode incidence at \( z=0 \) and no reflection from \( z=N_g L \), initial conditions are expressed as \( \mathbf{a}^D(0) = [1 \ 0 \ \cdots \ 0]^T \), \( \mathbf{a}^D(N_g L) = \mathbf{0} \). Substituting these conditions into (8), solutions \( \mathbf{a}^D(0) \) and \( \mathbf{a}^D(N_g L) \) are obtained. Then the reflected powers \( R_g, R_r \) and the transmitted powers \( T_g, T_r \) of the guided modes and the radiation fields can be expressed, respectively, as

\[
R_g = \sum_{k=1}^{K_1} |\mathbf{a}^D_k(0)|^2, \quad R_r = \sum_{k=K_1+1}^{2K_1} |\mathbf{a}^D_k(0)|^2, \quad T_g = \sum_{k=1}^{K_1} |\mathbf{a}^D_k(N_g L)|^2, \quad T_r = \sum_{k=K_1+1}^{2K_1} |\mathbf{a}^D_k(N_g L)|^2.
\]

(9)

Here \( T_g + T_r + R_g + R_r = 1 \), \( K_1 \) is the number of guided mode. The eigenvectors are normalized so that the total power of \( k \)-th mode carried in the \( z \)-direction is \( |\mathbf{a}^D_k|^2 \).

\[
\square \quad \text{NUMERICAL RESULTS}
\]

As the numerical examples in the present waveguides as shown in Fig.1, we choose

\[
\begin{align*}
K = 2b = 2b_1 &= 1.5 \ [\mu m], \ b_1 = 1.2 \ [\mu m] , \ b_1 - b_2 &= 1.3(b_1 - b_2), \ \Delta z = 0.25 \ [\mu m], \ \lambda_1 = \lambda_3 = 10\lambda, \ n_{21} = n_2 = 1.55, \ n_{22} = 1.50, \ n_1 = 1.45 \text{ and } M = N = 10.
\end{align*}
\]

In the case of sinusoidal surface relief which varies \( h' \cos(2\pi/L)z \), one period \( L \) of the guiding part is approximated by 12-layered thin rectangular plates, as shown in Fig.1(c), which is confirmed numerically to be enough in accuracy for qualitative comparison of the behavior on the radiation fields in each case of Fig.1.

In each waveguide, wavelength characteristics on reflected powers of dominant guided mode \( R_g \) and total radiation fields \( R_r \) are obtained from (9) for dominant mode incidence from \( z=0 \), and results are shown in Figs. 2-5.
First in the case of index-modulation type, radiation field $R_r$ increases largely as soon as the wavelength is shortened from the edge of main lobe of $R_{g1}$ (Fig.2) [2]. Then in normal use of the periodic waveguide as a filter, the suppression of this abnormal increase of $R_r$ is required. It has been confirmed in the case of semiconductor substrate as shown in Fig.2(c) that such a radiation field is almost completely suppressed, for example, by making the index also of the substrate part ($n_3$) in Fig.1(a) periodic, corresponding to the grating part formed by $n_{21}$ and $n_{22}$ [5]. In Fig.2(c), dotted curve shows the case of normal periodic waveguide as shown in Fig.1(a) which is quite similar to the case of glass substrate as shown in Fig.2(a), and solid curve shows the revised periodic waveguide in which the semiconductor substrate ($n_3 = 3.27$) is divided into two parts adding the layer of $n_{32} =$
3.17 periodically corresponding to \( n_{22} \). It can be seen from the solid curve in the figure that the abnormal large radiation field is clearly suppressed.

On the other hand, in the case of raised lamellar grating type with rectangular teeth as shown in Fig.1(b), \( R_r \) is considerably suppressed in shorter wavelength region and varies periodically, corresponding to the period of the side lobes of \( R_{g1} \), with almost constant peak value smaller than the maximum value of \( R_r \) within the main lobe of \( R_{g1} \) as shown in Fig.3(a) [3]. It is confirmed, however, that \( R_r \) becomes larger rapidly at much shorter wavelength (\( \lambda = 1.4677 \) [\( \mu \text{m} \]) than the case of index-modulation type (see Figs.2(b) and 3(b)), and one can not neglect such a large radiation field. In Fig.3(b) including Figs.4 and 5, the wavelength characteristics on side lobes of \( R_{g1} \) are omitted. In the case where the rectangular teeth of Fig.1(b) is replaced by approximated sinusoidal surface relief as shown in Fig.1(c), the magnitude of \( R_r \) increases a little than the case of Fig.3 as shown in Fig.4, because the magnitude of the groove depth \( 2h' \) is larger than the case of Fig.1(b) (\( b_1 - b_2 = 1.3(b_1 - b_2) \)), and also the large increase of \( R_r \) occurs at much shorter wavelength similarly as the case of Fig.3(b). Here we choose \( 2h' = 1.3(b_1 - b_2) \) so that the wavelength characteristics approach those of lamellar grating type with the rectangular teeth in Fig.3. It is noted that, if the depth \( 2h' \) is changed from \( 1.3(b_1 - b_2) \) the characteristics is also changed.

In the last case of embedded lamellar grating type where \( n_1 \) is exchanged for \( n_3 \) in Fig.1(b), that is, the corrugation part faces the substrate side. In this case, \( R_r \) can be suppressed remarkably and wavelength characteristics of the main lobe of \( R_{g1} \) is more flattened as shown in Fig.5 [4] than the case of raised lamellar grating type (Fig.3). This situation can be explained by the example obtained in the case of semiconductor substrate (\( n_2 = 3.5, n_3 = 3.15, n_1 = 1.0, 3.0 \)) as shown in

---

**Fig.4** Wavelength characteristics of \( R_{g1} \) and \( R_r \) in lamellar grating type periodic waveguide with sinusoidal surface relief of Fig.1(c)

**Fig.5** Wavelength characteristics of \( R_{g1} \) and \( R_r \) in the embedded lamellar grating type periodic waveguide in which index \( n_1 \) is exchanged for \( n_3 \) in Fig.1(b)

**Fig.6** Field distributions of reflected dominant guided mode for \( n_1 = 1.0 \) (raised lamellar grating) and \( n_1 = 3.0 \) (corresponds to embedded lamellar grating) in semiconductor substrate (\( n_2 = 3.5, n_3 = 3.15 \))
Fig.6. That is, the case of \( n_1 = 1 \) corresponds to the raised lamellar grating type (Fig.3(b)). On the other hand, in the case of \( n_1 = 3 \) which corresponds to the embedded one, the peak point of the field distribution of dominant guided mode shifts to the substrate side \( (n_1 = 3) \) and approaches the perturbation region. Then the power \( R_{\delta l} \) becomes larger by stronger coupling with the reflected space harmonic waves, and moreover the radiation field is suppressed largely due to the smaller difference of indices \( (n_2 - n_1) \). In the figure, when \( n_3 \) is moreover changed to 1.0, keeping \( n_1 = 3 \), the peak of field distribution more approaches the corrugation part and much stronger coupling is achieved (compare Figs.3(b) and 5). This case more approaches the example of Fig.5. However, in this case too, \( R_{\gamma} \) increases largely at much shorter wavelength \( (\lambda \sim 1.4645 \mu m) \) than the case of raised one.

In every case where the radiation field \( R_{\gamma} \) increases abnormally, the reflection of radiation field from the virtual periodic boundaries \( \lambda_x \) and \( \lambda_y \) can not be ignored for the accurate evaluation of the radiation field, and some absorbing layer surrounding the cross section of the domain considered is needed.

\section*{VI. CONCLUSION}

Applying the Fourier series expansion method, characteristic behaviors of the increase of reflected radiation field in the shorter wavelength region from the Bragg wavelength are made clear for four kinds of three dimensional periodic waveguides. That is, it is confirmed that, in the case of index-modulation type periodic waveguide, the reflected radiation field \( R_{\gamma} \) begins to increase largely as soon as the wavelength is shortened from the main lobe of reflected dominant guided mode \( R_{\delta l} \). These large radiation fields can be completely suppressed by making also the index of substrate part periodic similarly as the guiding part. On the other hand, in the case of lamellar grating type, \( R_{\gamma} \) is suppressed considerably and varies periodically in the shorter wavelength region with the magnitude smaller than the maximum value of \( R_{\gamma} \) within the main lobe of \( R_{\delta l} \), and the large increase of \( R_{\gamma} \) also appears at much shorter wavelength than the case of index-modulation type. In this case too, the radiation field can be much more suppressed only by making the corrugation part face the substrate side. These periodic waveguides in which radiation field in the shorter wavelength is strongly suppressed may be expected as a filter with flat wavelength characteristics. Accurate evaluation of the abnormally increased radiation field is a problem in future.

\section*{REFERENCES}