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MINIMIZATION OF THE RELATIVE CONVERGENCE PHENOMENON IN THE ANALYSIS OF FINLINE STEP DISCONTINUITIES USING THE WEIGHTING FUNCTIONS THAT INCORPORATE THE EDGE EFFECT IN THE GTR METHOD

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***Abstract*- In this paper a numerical study is carried out in order to establish a systematic convergence criteria for the Generalized Transverse Resonance (GTR) method. These criteria allow an optimum determination of the number of modes to be used in either side of a Finline Step Discontinuity under analysis. Applying the optimal modal ratio, and using basis functions incorporating the singular behavior of fields at edges and the efficient Singular Value Decomposition (SVD) technique, accurate results in the GTR method are obtained.**

Index Terms- Finline Step Discontinuity. Generalized Transverse Resonance Technique, Edge Effect.

I. INTRODUCTION

The GTR method used in this paper is studied in depth regarding the determination of the optimum modal ratio for given waveguide dimensions. This method leads to an ill-conditioned matrix which can be expressed in a homogeneous matrix equation of the form

$$[\mathrm{H}]\vec{\mathrm{X}} = \vec{0} \tag{1}$$

where [H] is a complex matrix of size (m,m) and $\vec{\mathrm{X}}$ is an m-element column vector. The elements of [H] are functions of a length parameter l_2 characteristic of the structure under analysis, Fig. 1.

In order to determine the solution of (1) we vary the length parameter l_2 until the determinant of [H] vanishes.
Instead of computing the determinant directly, we take the SVD and determine the minima of the minimum singular value as a function of l_2. We will show that this approach is superior.

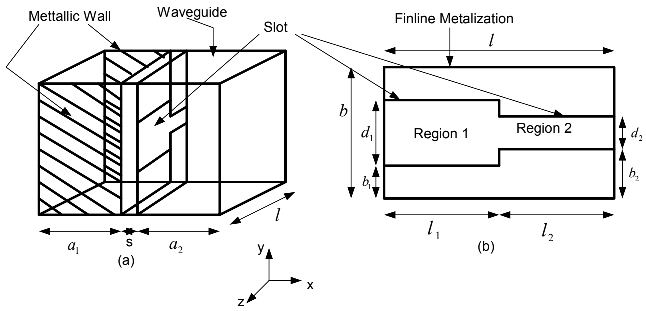

Fig. 1. Finline in a rectangular resonant cavity for the application of GTR method: (a) boxed finline step discontinuity (b) finline step discontinuity.

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II. PROBLEM FORMULATION

The application of the GTR method to a Step Finline Discontinuity Structure enclosed in a rectangular waveguide (fig. 1) needs the evaluation of inner products g between the orthonormal modal functions of the waveguide $\vec{e}$ and those of the slot $\vec{e}_0$.

$$g = \iint_S \vec{e} \cdot \vec{e}_0 ds \qquad (2)$$

where S is the surface of the slot enclosed in the rectangular cavity formed to use the GTR method. We introduce a double separable weighting function in the calculation of g which takes into account the edge effect of the field [1]:

$$\vec{e}'_0 = W_y W_z \vec{e}_0$$

$$g' = \iint_S \vec{e} \cdot \vec{e}'_0 ds = \iint_S \vec{e} \cdot \vec{e}_0 W_y W_z ds$$

$$W_{y\,(\text{region 1})} = \frac{1}{\sqrt{1 - \left[\left(y - \frac{d_2}{2}\right) \Big/ \left(\frac{d_1}{2} - \frac{d_2}{2}\right)\right]^2}} \qquad (3)$$

$$W_{y\,(region\,2)} = \frac{1}{\sqrt{1 - \left(y \Big/ \frac{d_2}{2}\right)^2}} \qquad W_z = \frac{1}{\sqrt{1 - (z / l_1)^2}}$$

This is equivalent to a generalization of the inner product. The electric field at the slot $\vec{e}'_0$ is given by the modal functions of an empty rectangular waveguide of width l_1 and height d_1 (region 1) and an empty rectangular waveguide of width l_2 and height d_2 (region 2) (figure 1.b) as done in [2].

The electric field in the waveguide region is given by the transverse components of the TE and TM modes in a rectangular empty waveguide of dimensions $l \cdot b$. In general

$$g = \int_0^{l_1} \int_{b_1}^{b_1+d_1} \vec{e}^{\,TE,TM} \cdot \vec{e}_{01}^{\,TE,TM} dydz + \int_{l_1}^{l} \int_{b_2}^{b_2+d_2} \vec{e}^{\,TE,TM} \cdot \vec{e}_{02}^{\,TE,TM} dydz$$

$$= f \left\{ \begin{array}{l} (m,n,r,s), (l_1,l_2,b_1,b_2,d_1,d_2), \\ J_0\left[\left(\frac{2n\pi}{b} + \frac{2r\pi}{d_1}\right)\left(\frac{d_1}{2} - \frac{d_2}{2}\right)\right], \\ H_0\left[\left(\frac{2n\pi}{b} + \frac{2r\pi}{d_1}\right)\left(\frac{d_1}{2} - \frac{d_2}{2}\right)\right] \end{array} \right\} \qquad (4)$$

The inner product g gives pure imaginary or pure real numbers which are function of modal indices (m,n,r,s), the physical dimensions (l,l_1,l_2,b_1,b_2,d_1,d_2), the Bessel function of first kind and order zero $J_0(x)$ and the Struve function $H_0(x)$ of order zero.

According to R. Mittra et al. [3], to fulfill the edge condition when analyzing the bifurcate problem for a rectangular waveguide with the mode matching method, the ratio of the number of modes used to represent the electromagnetic field in both zones (waveguide and aperture) of the structure should equal the heights ratio. This condition was proved by Y. C. Shih et al. [4] in the analysis of step discontinuities in rectangular waveguides. By extrapolating this result to the case of GTR method applied to a short-circuited finline it should yield a ratio

$$R = \frac{M}{Q} = \frac{b}{w} \qquad (5)$$

where M is the number of terms used to expand the fields in the waveguide zone, Q is the number of terms used to expand the fields in the aperture zone, b is the height of the waveguide, and w the height of the aperture. H. Hoffmann [5] got good and fast convergence using

$$R = \frac{M}{Q} = 1.5 \frac{b}{w} \qquad (6)$$

in the case of zero thickness [1].

In the case of the structure of Fig.1 we vary the length l_2 setting l_1 fixed and we search the resonant length l_2 that cancels the determinant of the matrix [H] in (1). Then, we evaluate the convergence of the method for different values of R in order to obtain the optimized R value.

III. NUMERICAL RESULTS

We started by determining the optimum value of w in (6). In our case, the height of the waveguide is b and the aperture is divided into two regions with heights d_1 and d_2 respectively

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(see Fig. 1.b). A function relating w to d_1 and d_2 is sought. For the sake of comparison we choose d_1 = 0.85 mm and d_2 = 0.25 mm. These are the same values that were used by R. Sorrentino et al. [2] and M. Helard et al. [6].

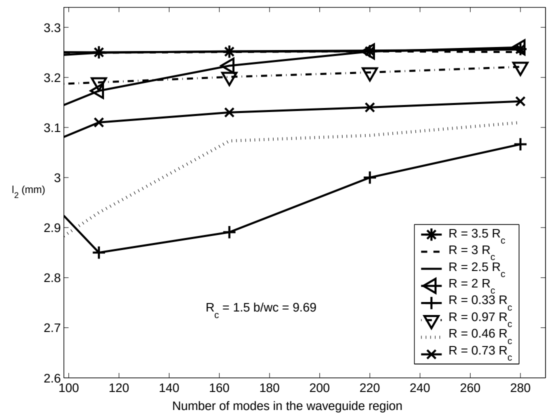

Fig. 2. Relative convergence phenomenon in the solution of the length (l_2) in a finline step discontinuity for different modal ratios (number of modes in the waveguide region divided by the number of modes in the slot region) R. Dimension as in Fig.1: l_1= 6.17 mm; a_2 = 3.3 mm; a_1 = b = 3.556 mm; d_1 = 0.85 mm; d_2= 0.25 mm; ε_r = 2.22; f = 30 GHz.

Fig.2 shows the evolution of the length l_2 in terms of the number of modes M when analyzing a finline step discontinuity. We can summarize the results as follows:

If we define $w_c = \dfrac{d_1 + d_2}{2}$ and

$$R_c = 1.5 \frac{b}{w_c} \tag{7}$$

a smooth convergence to the solution is obtained when

$$R = \frac{M}{Q} \geq R_c \tag{8}$$

For increased values of R, smaller values of M are sufficient for proper convergence. Note however that R cannot be made indefinitely large by reducing Q, since a minimum number of modes is necessary to reproduce accurately the field in the aperture. Also M cannot be indefinitely increased because the effect of roundup errors can yield completely inaccurate results.

To validate our l_2 result we compared the S parameters calculated with GTR (by determining three different l_2 resonance values corresponding to three different fixed l_1 values) with the S parameters calculated by R. Sorrentino et al. [2] and M. Helard et al. [6]. An excellent agreement is found as shown in fig. 3. The S parameters calculated with HFSS simulator by Ansoft, based on the finite element method (FEM) [7] are also presented.

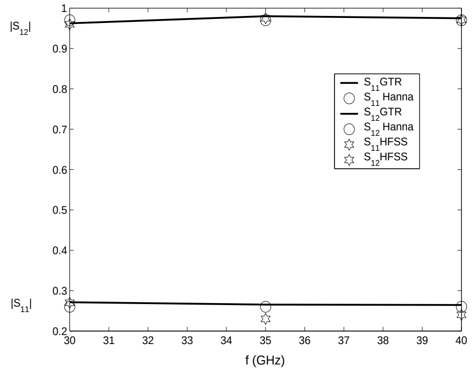

Fig. 3. Magnitude of scattering parameters of a unilateral finline step discontinuity for different frequencies

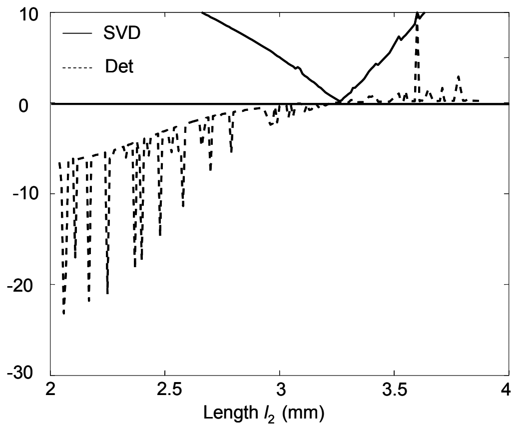

Fig.4. Behavior of the system determinant (dashed line) and minimum singular value (solid line) for a Modified Transverse Resonance formulation of a finline step discontinuity.

Table 1: Comparison of simulation time.

M	120	280
Q	10	26
t *(minutes)*	80.23	120.469
l_2 (mm)	3.25325	3.25105

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Figure 4 compares the determinant (dashed line) with the minimum singular value decomposition (solid line) versus the length l_2 in the case of the finline step discontinuity depicted in Figure 1.b. The zero of the determinant, which coincides with the occurrence of the minimum singular value, corresponds to the resonant length l_2 of the studied structure. The detection of minimum singular value v_m is easier and more accurate than obtaining the zeros of the determinant function. This is due to the existence of steep and abrupt changes in this function.

V. CONCLUSION

It has been found that fulfilling the condition given by (8) to fix the modal ratio allows an accurate determination of the resonant length for a boxed finline step discontinuity by using a small number of modes. This reduces drastically the CPU time. As an example, table 1 shows the time needed to obtain essentially the same result for two different pairs of *M* and *Q* number of modes when using a Matlab 6.5 code running on an Athlon processor based PC at 1.67 GHz. The use of the SVD method overcomes efficiently the numerical problems related with the ill-condition of the resulting matrices related with the homogenous system (1). This technique allows avoiding the poles and gradients that are common in the determinant function and improves the accuracy of the computer results.

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