A Simple Approach to Control the Time-constant of Microwave Integrators

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Abstract- In this paper, firstly the time-constant of an integrator is described in continuous as well as in the discrete-time domains. Further, a simple relation is expressed to find the time-constant of an integrator from continuous-time domain to discrete-time domain or vice-versa. Thereafter, the microwave integrators of different time-constants are obtained by optimizing the characteristic impedance of an open-circuited shunt stub using linear programming approach in z-domain. It is clear that all the microwave integrators of different time-constants are realized only by using a single open-circuited shunt stub.

Index Terms- Digital integrator, microwave integrator, open-circuited shunt stub, optimization, time-constant.

I. INTRODUCTION

The integrators are very useful systems to estimate the time integrals of measured signals. These systems have a lot of applications for waveform shaping, accumulator analysis, coherent detection, control systems etc. It is noticed that the integrators are generally designed for systems of low-frequency applications and rarely for high-frequency or microwave applications. The frequency response of an ideal integrator is defined by

\[ H_i(j\omega) = \frac{K}{j\omega} \quad (1) \]

Where \( j = \sqrt{-1} \), \( K \) is a scaling constant and \( \omega \) is the angular frequency in radians per second. A series resistor-capacitor (RC) circuit, in conjunction with an operational amplifier, has been generally used to design an integrator.

However, such type of integrator is useful only for low-frequency applications.

It is interesting to note that a lot of research work is available in the area of digital integrators using recursive systems. Initially, the recursive digital integrators have been designed by taking the Z-transform of existing integration rules. Further, these are obtained by performing a simple linear interpolation between the magnitude responses of classical rectangular, trapezoidal and Simpson digital integrators [1-3]. Thereafter, a linear programming optimization approach has been proposed to design recursive digital integrators [4]. In [5], Ngo has proposed a recursive wideband digital integrator of third-order system based on Newton-Cotes integration rule. Further, Gupta-Jain-Kumar have proposed recursive wideband digital integrators for comparatively lower percentage relative errors (PREs) in magnitude responses over wideband with the ideal integrator [6]. Later, Al-Alaoui has also proposed a novel class of 2-segment, optimized 3-segment and optimized 4-segment recursive digital integrators [7]. Recently, Upadhyay et al. have designed the recursive wideband digital integrators for lower relative errors in magnitude responses by using coefficient and pole-zero optimizations [8-10].

It is noted that the digital integrators cannot be realized practically in the frequency range of GHz or microwave range by using the adder, multiplier and delay circuits. Therefore, the designing of microwave integrators is the emerging field in current research environment.
In [11], Hsue-Tsai-Kan have designed the first-order trapezoidal rule based microwave integrator. Further, Hsue-Tsai-Tsai have proposed the time-constant control of microwave integrators by cascading the different number of transmission line sections [12]. The combination of time-constant and magnitude response describe the system behavior of an integrator. Later, Tsai-Fang have designed the second-order microwave integrators by using two section open-circuited shunt stubs and the serial transmission line sections [13].

It is noticed that the existing Hsue-Tsai-Tsai (HTT) approach for controlling the time-constant of microwave integrators is not a simple approach [12]. This approach requires auto regression moving average (ARMA) process in z-domain. Further, this approach has the need of six to ten transmission line sections to design microwave integrators of required time-constants. The sizes of microstrips Layouts for microwave integrators of different time-constants are approximately from 4.5 cm to 8.5 cm [12]. Therefore, there is the need to develop a simple approach for controlling the time-constant of microwave integrators with the constraint of compact design formulation.

In this paper, a simple approach is proposed to control the time-constant of microwave integrators. All the designed integrators of different time-constants require a transmission line i.e. shunted with a single open-circuited stub.

Section II describes the time-constant of an integrator. This section also provides a relation to relate the time-constant of an integrator in continuous-time and discrete-time domains. Further, analysis of microwave integrator and time-constant control are given in section III. Thereafter, results are discussed in section IV. Finally, conclusions are given in section V.

II. TIME-CONSTANT OF AN INTEGRATOR

Several techniques have been developed to design recursive digital integrators in the study of digital signal processing (DSP). In [12], Hsue-Tsai-Tsai (HTT) have designed a new recursive digital integrator by applying the linear interpolation over trapezoidal integration rule and the inverse of a wide-band Hsue-Tsai-Chen (HTC) differentiator [14]. The transfer function of HTT integrator is given below.

$$H_i(z) = \frac{0.26 + 0.073z^{-1}}{1 - z^{-1}}$$  \hspace{1cm} (2)

Fig.1 shows the magnitude response of HTT digital integrator with the ideal integrator of time-constant $\tau_{di} = 3.18$ over the full Nyquist band.

In [12], HTT have also defined the time-constant ($\tau_{wi}$) of an integrator by $1/|H(j\Omega)|\Omega$ in continuous-time domain, where $H(j\Omega)$ is the transfer function of an integrator in frequency domain and $\Omega$ is signal angular frequency. Similarly, the time-constant ($\tau_{di}$) of an integrator can also be defined by $1/|H(e^{j\omega})|\omega$ in discrete-time domain, where $H(e^{j\omega})$ is the
transfer function of digital integrator and $\omega$ is the normalized frequency of discrete-time signal. Now the time-constant of HTT digital integrator is determined at particular normalized frequency using Fig. 1. The procedure to determine the time-constant is given below.

$$\tau_{di} = \frac{1}{|H(e^{j\omega})|\omega} = \frac{1}{0.3114(0.3215\pi)} \approx 3.18$$ (3)

It is clear in Eq. 3 that the time-constant of HTT integrator is 3.18 for normalized frequency $\omega=0.3215\pi$. The percentage relative error (PRE) of any digital integrator in magnitude response with the ideal integrator of corresponding time-constant is defined by

$$PRE = \frac{100 \times \left| H(e^{j\omega}) \right| - \frac{1}{\omega\tau_{di}}}{\frac{1}{\omega\tau_{di}}}$$ (4)

Fig. 2 shows the PRE of existing HTT digital integrator in magnitude response over the full Nyquist band with the ideal integrator of time-constant $\tau_{di} = 3.18$.

From Fig.2, it is observed that the HTT integrator has maximum PRE of 13.86% in magnitude response over the full Nyquist band with the ideal integrator of time-constant $\tau_{di} = 3.18$.

Now a simple relation is expressed to convert the time-constants of an integrator from discrete-time to continuous-time domain or vice-versa as in [15]. The generalized relation is defined as

$$\tau_{ct} = \frac{1}{|H_j\left(j2\pi f\right)|2\pi f}$$

$$= \frac{1}{Y(magnitude)\left[2\pi X(frequency)\right]} = \frac{\tau_{di}}{2f_0}$$

Where $f_0$ is the normalizing or maximum operating frequency. If $f_0= 10$ GHz, then the time-constant of existing HTT digital integrator ($\tau_{di} = 3.18$) will be 0.159 ns in continuous-time domain.

III. ANALYSIS OF MICROWAVE INTTEGRATOR AND TIME CONSTANT CONTROL

In [16, 17], the chain scattering parameters of a transmission line i.e., shunted with an open-circuited stub is defined in the z-domain. These parameters are given below in matrix form.

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \frac{1}{1+z^{-1}} \begin{bmatrix} (1+c')(1-c')z^{-1} & c'(1-z^{-1}) \\ -c'z^{-1} & (1-c')+(1+c')z^{-1} \end{bmatrix}$$ (6)

Therefore, the transfer function of a transmission line i.e., shunted with an open-circuited stub can be written as

$$T_m(z) = \frac{1}{T_{11}(z)} = \frac{1+z^{-1}}{(1+c')+(1-c')z^{-1}}$$ (7)

Where $c'=Z_0/(2Z_o)$; $Z_a$ is the characteristic impedance of open-circuited shunt stub, $Z_0$ is the characteristic impedance of main transmission line.
It is noticed that the system function $T_n(z)$ behaves as a low-pass filter. Therefore, the microwave integrator of required time-constant ($\tau_{ai}$) in continuous-time domain or $\tau_{di} = 2f_0\tau_{ai}$ in discrete-time domain may be designed by using a transmission line i.e. shunted with an open-circuited stub. The $PRE$ of a general open-circuited shunt stub in magnitude response with the ideal integrator of required time-constant ($\tau_{di} = 2f_0\tau_{ai}$) is defined by

$$PRE = 100 \times \left[ \frac{T_r(e^{j\omega})}{\omega \tau_{di}} \right]$$

(8)

Now, the microwave integrator of required time-constant ($\tau_{ai}$) is obtained by minimizing the maximum $PRE$ over specific Nyquist band using the linear programming approach [4]. In this minimization, $c'$ is used as the optimization parameter. In particular, the time-constant of 0.053 ns ($\tau_{ai} = 1.06$, if $f_0 = 10$ GHz) is achieved for the value of optimized parameter $c' = 1.213$ over a specific Nyquist band. The transfer function of proposed microwave integrator for the time-constant of 0.053 ns ($\tau_{ai} = 1.06$), thus obtained is given in (9).

$$T_i(z) = \frac{1 + z^{-1}}{2.213 - 0.213z^{-1}}$$

(9)

Notice that $Z_a$ is 20.61 Ω, if $Z_0$ is 50 Ω. This reveals that a transmission line i.e., shunted with an open-circuited stub of $Z_a = 20.61$ Ω and length $L_0 = \frac{\lambda_0}{4}$ (where $\lambda_0$ is the wavelength i.e. corresponding to the normalizing or maximum operating frequency of 10 GHz) can be employed to design a microwave integrator of time-constant ($\tau_{ai} = 0.053$ ns) over a specific operating band. This approach is also applied to design microwave integrators of different time-constants. Table I shows the suitable values of optimized parameters ($c'$) or characteristic impedances ($Z_a$) of open-circuited shunt stubs for designed microwave integrators of different time-constants ($\tau_{ai}$). Fig. 3 shows the design formulation of microwave integrators for different time-constants.

Table 1: Suitable values of characteristic impedances of open-circuited stubs for designed microwave integrators of different time-constants

<table>
<thead>
<tr>
<th>Microwave integrators</th>
<th>TC4</th>
<th>TC5</th>
<th>TC7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{ai}$ in ns</td>
<td>0.053</td>
<td>0.0795</td>
<td>0.159</td>
</tr>
<tr>
<td>$\tau_{ai} = 2f_0\tau_{ai}$, $f_0 = 10$ GHz</td>
<td>1.06</td>
<td>1.59</td>
<td>3.18</td>
</tr>
<tr>
<td>Optimum value of $c'$</td>
<td>1.213</td>
<td>2.202</td>
<td>5.19</td>
</tr>
<tr>
<td>$Z_a$ in Ω for $Z_a = 50$ Ω</td>
<td>20.61</td>
<td>11.36</td>
<td>4.82</td>
</tr>
</tbody>
</table>

Fig.3. Proposed design formulation of microwave integrators

Now, the transfer function $S_{21}(f)$ of proposed design formulation of microwave integrators is obtained in terms of the optimized parameter ($c'$) and the physical length of open-circuited stub ($L$) by using the transmission line parameters concept [18-20]. The transmission line matrix for open-circuited shunt stub is defined as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ i\tan(\beta L) \end{bmatrix} \begin{bmatrix} 0 \\ Z_a \end{bmatrix}$$

(10)
The scattering parameter $S_{21}$ of any system relates with the transmission line parameters as

$$S_{21} = \frac{2}{A + (B/Z_0) + C Z_0 + D} \quad (11)$$

Therefore, the system function $S_{21}$ of proposed design formulation for microwave integrator thus obtained is given below.

$$S_{21}(f) = \frac{1}{1 + j c' \tan \left( \frac{2\pi f \sqrt{\varepsilon_{eff} L}}{c} \right)} \quad (12)$$

Where $c' = Z_0 / (2Z_a)$, $c$ is the velocity of light, $f$ is the operating frequency, $L$ is the physical length of open-circuited shunt stub and $\varepsilon_{eff}$ is the effective dielectric constant of microstrips structure. Let the physical length of microstrips structure is $L = \lambda_0 / (4\sqrt{\varepsilon_{eff}})$ (where $\lambda_0$ is the wavelength and it is corresponding to the normalizing frequency of 10 GHz).

IV. RESULTS AND DISCUSSION

Fig. 4 shows the magnitude responses of designed microwave integrators TC4, TC5 and TC7 in z-domain with the ideal integrators of required time-constants ($\tau_{di}$) as 0.6, 1.06 and 1.59, respectively. Further, Fig. 5 shows the PREs of designed microwave integrators in magnitude responses with the ideal integrators of required time-constants ($\tau_{di}$) as 0.6, 1.06 and 1.59, respectively.

From Figs. 4 and 5, it is observed that the designed integrators TC4, TC5 and TC7 have not more than 6% relative error in magnitude responses over the specific Nyquist bands. Further, it is observed that the larger time-constant integrator design (TC7) has more Nyquist bandwidth as compared to the smaller time-constant integrator design (TC4). It is also interesting to note that the larger time-constant integrator design is more preferable in lower band as compared to the smaller time-constant integrator design.

Fig. 6 shows the magnitude responses of $S_{21}(f)$ for designed TC4, TC5 and TC7 microwave integrators with the ideal ones of corresponding time-constants ($\tau_{di}$). Further Fig. 7 shows the
PREs of these designed integrators with the ideal ones of corresponding time-constants (τai).

From Figs. 6 and 7, it is observed that the designed TC4, TC5 and TC7 microwave integrators have not more than 6% relative error in magnitude responses over the specific operating bands. Further, it is observed that the TC4 microwave integrator has the time-constant of 0.53 x 10^{-10} second over the frequency range of 3.4 GHz to 6.7 GHz, the TC5 integrator has the time-constant of 0.795 x 10^{-10} second over the frequency range of 2.6 GHz to 6.0 GHz and the TC7 microwave integrator has the time-constant of 1.59 x 10^{-10} second over the frequency range of 1.5 GHz to 5.0 GHz. Therefore, it is clear that the microwave integrator of larger time-constant has more operating bandwidth as compared to the microwave integrator of smaller time-constant. It is interesting to note that the larger time-constant integrator design (TC7) is useful in lower band as compared to the smaller time-constant integrator design. It is also clear that the microwave integrator of required time-constant for band limited applications can be easily designed by using the single open-circuited shunt stub.

V. CONCLUSION

A simple approach is presented to control the time-constant of microwave integrators. This approach is much simpler in comparison with the existing approach and it requires a single transmission line element i.e. shunted with an open-circuited stub to control the time-constant. It is shown that the microwave integrator of larger time-constant has more operating bandwidth as compared to the integrator of smaller time-constant. The main drawback of proposed approach is the smaller operating bandwidth for smaller time-constant. However, the proposed approach is preferable for larger time-constant over lower-band.

ACKNOWLEDGMENT

The authors are highly grateful to Prof. Raj Senani (Director, NSIT Dwarka, New Delhi, India) and Prof. D. S. Chauhan (Vice-Chancellor, UTU Dehradun, India) for encouraging the research activities.
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