Short-Circuited Quarter Wavelength Cylindrical-Rectangular Microstrip Patch Antenna

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Abstract: The objective of this work is to develop a simple method to calculate the input impedance and radiation pattern of short-circuited quarter wavelength long cylindrical-rectangular microstrip patch antenna. It is also to study the effect of the size of cylinder as well as the parameters of dielectric substrate. In this study we would like to take into account the problems of resonance and radiation of a short-circuited quarter wavelength cylindrical-rectangular microstrip patch antenna. Transmission line model is used to calculate the input impedance of the patch, while the combination of array theory and cavity model is used to calculate its radiation pattern. The volume covered by electric and magnetic walls change with radius of the magnetic plane. It is observed that the beam-width, resonant frequency, and resonant resistance decrease with cylinder radius. Peak radiated power increases with cylinder radius, but beyond the critical angle, radiated power decreases with cylinder radius. Critical angle increases with cylinder radius and substrate permittivity. It is also observed that difference of peak radiated power decreases with substrate permittivity, and power radiation beyond 90° is possible for cylindrical rectangular patch. The bandwidth is not sensitive to curvature but it decreases as substrate permittivity increases.

Index Terms: Compact microstrip antenna, Cylindrical-rectangular patch, Shorted wall, Quarter wave patch.

I INTRODUCTION

One of the major advantages of microstrip antennas is that they can be made conformal to the surfaces on which they are mounted because of their low profile. Most current investigations of microstrip antennas have been concentrated on planar structures. However, often this plane surface is either distorted or antenna elements are intentionally placed on a curved surface. Cylindrical microstrip antennas find many applications pertaining to high-speed aircrafts, because of their conformability with the aerodynamical structure of such vehicles.

In recent research work, much attention has been focused on the development of small antennas on a cylindrical surface for applications where limited antenna real state is available, which is important in many commercial and military applications.

Among several small antennas, the shorted microstrip patch has been a popular candidate [1]. The resonant frequency of a microstrip antenna can be significantly reduced by introducing a short-circuited plane or a partly short-circuited plane where the electric field of the resonant mode is zero. So the length of the patch is half of the original length and resonant frequency of the patch remains unchanged [2]. While such antennas have been used in many applications, basic studies of the effect of curvature are rather scarce [3]-[8].
Some work has been devoted to the analysis of microstrip patch antenna mounted on cylindrical surface. This structure was first proposed by Krowne [1]. Using a cavity model, he observed that resonant frequency changes with surface curvature. Wu and Kaufman [3], calculated the radiation patterns using cavity model in conjunction with the method of images, but this method is not applicable when the ground plane is not flat. In the paper by Fonseca and Giarola [4], the radiation from the wraparound cylindrical microstrip element was computed from a magnetic wall cavity model. In the paper by Ashkenazy et al. [6], the radiation from the wraparound and the rectangular patches was computed by assuming an electric surface current distribution on microstrip patch antenna. Ashkenazy et al. [6], show an analysis of microstrip antennas on cylindrical substrates for a given current distribution on the patches. Dahele et al. [7], investigated the effect of curvature on the characteristics of rectangular patch antenna theoretically and experimentally. They found that for $TM_{01}$ mode, the resonant frequency is not affected by curvature. However, as curvature increases the pattern broadens, the resonant resistance decreases, and bandwidth increases. Luk et al. [9], considered the case when the substrate thickness is much smaller than the wavelength and the radius of curvature. Based on the cavity model, they found that the resonant frequencies and electric field under the patch were not affected by curvature. However, the patterns, Q factors, and input impedances are affected. Habashy et al. [10], calculated the input impedance and radiated field from the cylindrical-rectangular and the wraparound elements excited by a probe using moment method.

The moment method has been widely used for the calculation of the input impedance of microstrip patch antennas fed by means of probe on cylindrical substrate. More recently, full wave approach was applied to microstrip patch antennas [10]-[11]. In those works, only the single rectangular patch was studied. The method of moments [12] and the general transmission line model have been used to analyze microstrip patch antennas on spherical and cylindrical supports. Wong and Ke [13] investigated the curvature effect on aspect ratio and circular polarization conditions for cylindrical-rectangular microstrip patch antennas. Kashiwa et al. [14], calculated the rectangular microstrip patch antennas mounted on the curved surface wave analyzed using curvilinear FD-TD method. Since a microstrip patch antenna is a highly resonant structure, its current distribution and input impedance at resonance are mainly determined by internal structures such as the shape of the patch, the thickness, and the dielectric constant of the substrate and superstrate. Provided that a patch antenna is placed on a locally flat surface or surface with small curvature, the shape of the host cylinder (external structure) has little effect on its current distribution and input impedance, as demonstrated in Kempel [15]. Tam et al [16], calculated the mutual coupling for a probe fed cylindrical-rectangular microstrip array using Green's functions which incorporate the effect of the size of cylinder as well as the parameters of dielectric substrate. Mutual coupling between two rectangular microstrip antennas on cylindrical surface has been studied by [17] – [20]. Mutual coupling between two triangular microstrip patch antennas mounted on a cylindrical surface was first calculated by Pan and Wang [21]. Zaid et al. [22], investigated the input impedance, the resonant frequency, the current density maps on the surface of each resonator and far field radiation patterns of stacked quarter wavelength microstrip patch antenna. The different radiating elements considered are end-shorted along one edge. An end-shorted element presents the advantage of the smallest size with a length of $\lambda/4$, a broad beamwidth and a bandwidth which is reduced compared with a half element. Gue et al. [23], presented a compact dual-band patch antenna design using slot-loaded and short-circuited size reduction techniques.

Thus to determine the correct model field solution to the electromagnetic cavity problem, which can be used to find the radiation field solution, this curvature should be taken into account. The objective of this work is to develop a simple method to calculate the input impedance, VSWR, and radiation pattern of short-circuited quarter wavelength long
cylindrical-rectangular microstrip patch antenna. It is also to study the effect of the size of cylinder as well as the parameters of dielectric substrate.

II THEORETICAL FORMULATION

In this paper we report on the resonance and radiation problems of a short-circuited quarter wavelength cylindrical-rectangular microstrip patch antenna (CRMPA). Transmission line model is used to calculate the input impedance of the patch, while the combination of array theory and cavity model is used to calculate its radiation pattern.

A. Calculation of Effective Length and Width of the Patch

The geometry of a short-circuited quarter wavelength cylindrical-rectangular microstrip patch antenna is shown in Fig. 1. The straight edge of the patch has a dimension of \( L = \frac{\lambda}{4} \) and the curved edge has a length of \( W' = 2(R + h)\psi \), where \( 2\psi \) is the angle subtended by the curved patch and \( R \) is cylinder radius. The width of the patch may be find as [24]

\[
W' = \frac{c}{2f_d} \left( \frac{\varepsilon_r + 1}{2} \right)^{\frac{1}{2}}
\]

(1)

where \( \varepsilon_r \) is relative permittivity of substrate and \( f_d \) is designed frequency of antenna.

From Fig. 2, it is clear that the volume covered by electric and magnetic walls change with radius of the magnetic plane, which results the change in the effective dimensions of the patch. So the effective width of the patch may be find as

\[
W = \frac{R + h/2}{R + h} \cdot W'
\]

(2)

and the effective length of the patch may be calculate as [24]

\[
L = \frac{c}{4f_d \sqrt{\varepsilon_{eff}}} - \Delta l
\]

(3)

Where \( C \) is speed of light in free space and \( \Delta l \) is the fringing field length.

Fig. 1: A short-circuited rectangular patch mounted over cylindrical surface.

Fig. 2: Effective width.
\[ \Delta l = 0.412h \left( \frac{\varepsilon_{\text{eff}} + 0.3}{\varepsilon_{\text{eff}} - 0.258} \right) \left( \frac{W}{h} + 0.264 \right) \left( \frac{W}{h} + 0.8 \right) \]  

(4)

where \( h \) is substrate thickness and \( \varepsilon_{\text{eff}} \) is the effective relative permittivity of the substrate material.

The patch impedance at any point \((x_0, y_0)\) may be given as [24]

\[ R_{\text{rad}} = R_{\text{rad}}^p \cos^2 \left( \frac{\pi y_0}{L} \right) \]  

(6)

where \( R_{\text{rad}}^e \) = radiation resistance of feed probe.

This formula is valid on the condition that the feed position \( y_0 \) is located along L side and \( x_0 \) is at \( W/2 \) (Fig. 1), and \( R_{\text{rad}}^p \) = edge fed resonant resistance of the patch, which is given by [24]

\[ R_{\text{rad}}^p = 90 \frac{\lambda_0^2}{W^2} \text{ for } W<<\lambda_0 \]

\[ R_{\text{rad}}^p = 120 \frac{\lambda_0}{W} \text{ for } W>>\lambda_0 \]  

(7)

C. Calculation of Resonant Frequency

For the case of \( h \) much smaller than one wavelength, the cavity model or modal-expansion approximation can be adopted for analyzing the patch antenna on a thin substrate. In this case the resonant frequencies of the \( TM_{mn} \) modes for the cylindrical-rectangular microstrip patch antenna under the additional condition \( h<<R \), are given as [24]

\[ f_{mn} = \frac{c}{2\sqrt{\varepsilon_r}} \left[ \left( \frac{m}{2(R+h)\varepsilon_r} \right)^2 + \left( \frac{n}{L} \right)^2 \right]^{1/2} \]  

(8)

Equation (7) shows that if the dimension of the patch, i.e., \( 2(R+h)\varepsilon_r \) and \( L \), are fixed, the resonant frequencies of \( TM_{mn} \) modes are not affected by curvature. This conclusion is valid for thin substrate satisfying \( h<<R \).

D. Calculation of Input Impedance

From Fig. 3, the input impedance offered by the patch \( (Z_{in}) \) may be given by

\[ Z_{in} = j\omega L_p + Z_{in1} \parallel Z_{in2} \]  

(9)

where \( \omega \) is operating angular frequency, and \( L_p \) is self inductance offered by the probe, and \( Z_{in1} \) and \( Z_{in2} \) are the impedance offered by the right and left hand section of T. L. respectively.

In computing the self-impedance of the probe, the nonzero radius of the probe has to be considered. However, this will lead to a complicated expression for the self-impedance in the cylindrically stratified medium case. An approximate expression has to be computed, by neglecting the curvature of the cylinder structure and assuming the probe to be embedded in a planar stratified medium. This is a reasonable approximation if the radius of curvature of the cylindrical structure is sufficiently large as compared to the substrate thickness and the operating wavelength [9].

\[ \lambda_0/4 \]

Fig. 3: Equivalent circuit of short-circuited rectangular patch mounted over cylindrical surface.
From transmission line theory

\[ Z_{in1} = Z_o \left( G + jB \right) + jZ_o \tan(\beta y_o) \]

and

\[ Z_{in2} = jZ_o \tan \left( \frac{\lambda}{4} - y_o \right) \]  
(10)

Where \( G \) and \( B \) are conductance and susceptance of fringing field respectively, \( Z_o \) is characteristic impedance of patch as transmission line, and \( \beta \) is phase constant.

E. Radiation Pattern

In order to calculate the far-zone fields, the probe to be a \( \rho \) directed unit-amplitude current ribbon has been modeled [Fig. 4]. In this case the electric fields under the curved patch have only \( E_{\rho} \) component, which is independent of \( \rho \). From Fig. 4, consider the slot as an antenna array where all element of the array are in same phase and of equal amplitude but their alignment is followed by the curvature of the host surface. Because these array elements are not point source, therefore they behave as non-isotropic elements, leading an additional factor to come into existence \( \sin \alpha \), where \( \alpha \) is the inclination angle of far-field point from the plane normal to the element and passing through its center. From Fig. 4, \( \alpha \) may be determined as \( \alpha = 90 - (\phi - \phi) \). Using the array theory, total far-field electric field may be written as

\[ E_i = E_1 e^{j(\delta_1 + \beta d_1)} \cos(\phi - \theta_1) \]

\[ + E_2 e^{j(\delta_2 + \beta d_2)} \cos(\phi - \theta_2) \]

\[ + \ldots + E_n e^{j(\delta_n + \beta d_n)} \cos(\phi - \theta_n) \]  
(11)

where \( E_n \) is electric field strength of \( n^{th} \) element, \( \delta_n \) is phase difference of the \( n^{th} \) source element with reference source, and \( d_n \) is the path difference of the \( n^{th} \) element with reference source, \( \phi \) is the angle of reference line from vertical axis, and \( \theta_n \) is the angle of \( n^{th} \) element line from vertical axis. Here

\[ E_1 = E_2 = \ldots = E_n = E_o \]

\( \delta_1 = \delta_2 = \ldots = \delta_n = 0 \), and

\[ d_n = R [1 - \cos(\phi - \theta_n)] \]  
(12)

But, only that part of the curved patch will contribute on radiation, which face to far-field point and rest will not because ground of the patch reflect the field. Substituting the above values in (4), far-field field may be written as

\[ E_i = E_o e^{-j\beta R} \int_{\theta_L}^{\theta_R} e^{-j\beta R \cos(\phi - \theta)} \cos(\phi - \theta) d\theta \]

(13)

where \( \theta_L \) and \( \theta_R \) are the left most and right most inclination angle of the elements from the reference line which contribute on radiation and
Fig. 5(a)-(d): Variation of radiated power for different substrate permittivity and cylinder radii.
Fig. 6(a)-(d): Variation of impedance with frequency for different substrate permittivity and cylinder radii.
\( E_o \) is the voltage across the slot and may be given as [24]

\[
E_o = \begin{cases} 
\frac{W_s}{k^2 LW} + \\
\sum_{n=1}^{\infty} \frac{2W_s}{LW \left( k^2 - K_{on}^2 \right)} \cos \left( \frac{n \pi}{W} y \right) + \\
\sum_{m=1}^{\infty} \frac{4R_m}{m \pi W \left( k^2 - K_{mo}^2 \right)} \cos \left( \frac{m \pi}{L} y \right) + \\
\sum_{m=1}^{\infty} \frac{8R_m}{m \pi W \left( k^2 - K_{mn}^2 \right)} \cos \left( \frac{m \pi}{L} x \right) 
\end{cases}
\]

(14)

### III RESULTS AND DISCUSSION

From Fig. 5(a)-(d) it is observed that (a) the beamwidth increases with radius of curvature (b) peak radiated power increases with radius of curvature (c) it is observed that beyond the critical angle, radiated power decreases with radius of curvature and increases with radius of curvature for angle less than critical angle (d) power radiation beyond 90° is possible for cylindrical rectangular patch.

From Fig. 6(a)-(d) it is observed that the resonant resistance increases as radius of curvature decreases for different substrate permittivities. It is also observed that resonant frequency increases as radius of curvature decreases, but the change in the resonant frequency is not significant (maximum change is 0.3%). Almost same results were observed by Wong & Ke [15] for cylindrical-rectangular patch. The bandwidth is not sensitive to curvature but it decreases as substrate permittivity increases.

### IV REFERENCES


