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Analysis of Coupled Microstrip Lines Separated by an Embedded Metamaterial Region Using Dispersive FDTD Method

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***Abstract*-Full-wave analysis of metamaterial based microwave circuit is performed using two different FDTD formulations for dispersive medium. Both Mobius-transform technique and hybrid Mobius-ADE algorithm are employed to model media characterized by negative permittivity (Drude Model) and negative permeability (Lorenz Model) over a certain frequency range. The formulations are used to determine the effect of a homogeneous isotropic metamaterial slab sandwiched between two coupled microstrip lines on forward and backward coupling. Similar observations are obtained for both full-wave analysis techniques, which indicate reduction in forward coupling over the range where the slab behaves as a Double Negative (DNG) media. A parallel version of the developed algorithm is also tested with reduction in computational effort.**

***Index Terms*- Dispersive FDTD, metamaterial, effective medium approach, Mobius Transform, ADE, and coupled microstrip lines.**

I. INTRODUCTION

Metamaterials and its application to microwave circuits is a rigorously researched area for little over a decade with the ignition coming from initial work by Smith et al. [1]. It was shown by Pendry *et al.* [2] that, artificially negative effective permeability can be realized using conglomeration of periodically spaced sub-wavelength structures like split ring resonators (SRR). Likewise an array of wires can effectively produce negative permittivity. A unit cell as well as an array of SRR and wire in planar domain is illustrated in Fig. 1. With the development of metamaterials the design of coupled lines in conjunction with metamaterials saw renewed interest among microwave engineers. Enhanced coupling was reported by Itoh et al. [3] for couplers. Some design aspects of metamaterial inclusion focused on improved coupling for mode splitting hence resulting in dual band coupler design [4]. Though most of the initial application was easily analyzed using equivalent circuit models but full-wave analysis of complex array of these sub-wavelength structures in conjunction with microwave circuit and antenna posed a computation challenge. The obvious reason behind this is that the mesh size of the minute metamaterial particles resulted in huge meshing in the entire problem domain that often includes few wavelengths size circuits. However in the FDTD method, wave propagation in dispersive materials have been solved using recursive convolution [5], auxiliary differential equations (ADE) [5], z-transform [5] and Mobius transform [6] methods as applied to the Drude and Lorentz model. Incorporating piecewise linear recursive convolution (PLRC) for Lorenz media and simple treatment using auxiliary variables of Drude media, effective medium simulation of wave propagation in metamaterials was reported in [7]. It is discussed in [7] that PLRC could not be applied to Drude model because of the difficulty to obtain inverse Fourier transform due to presence of poles in the susceptibility of the effective permittivity function on the imaginary axis. This problem was removed in [8] and PLRC was applied to model both the effective media. A unified FDTD approach was proposed in [9] for general dispersive media where coordinate stretched Maxwell's curl equation in time domain was first deduced and then FDTD update formulas were combined with the semi-analytical recursive convolution (SARC) using Digital Signal Processing (DSP) technique. A higher order recursive convolution scheme based on trapezoidal rule was reported in [10].

In this work dispersive medium property of left-handed material is modeled using Mobius FDTD and a hybrid of Mobius as well as auxiliary differential equation (ADE) FDTD. These two different dispersive FDTD strategies improve computational efficiency and removes complexity in use of PLRC technique in modeling bulk effective metamaterial medium in conjunction with microwave circuits. The formulation is tested for the case of a DNG metamaterial slab is sandwiched between a pair of coupled microstrip lines as illustrated in Fig. 2. The Drude model for negative permittivity is formulated by Mobius transform method and Lorenz model for negative permeability is implemented by ADE as well as Mobius transform technique. Apart from the use of the hybrid-model for metamaterials, this work introduces vectorized-FDTD algorithm which gives considerable speed-up compared to the serial FDTD code. The full wave analysis shows enhancement of forward coupling and reduction of backward coupling between coupled microstrip lines due to inclusion of metamaterials.

The rest of the paper is organized as follows. Section-II deals with the mathematical formulation of the Mobius-FDTD algorithm for analysis of DNG media. Section-III presents the results which illustrate the effect of a DNG metamaterial slab on forward and backward

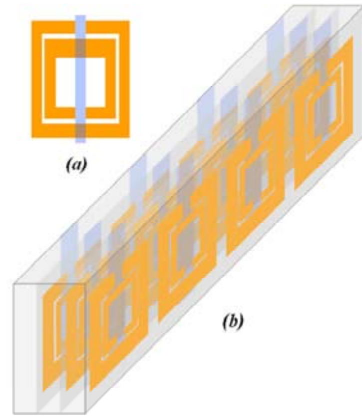

Fig. 1. Illustration of unit cell of SRR and wire and a block of DNG media formed by an array of these sub-wavelength structures. (a) 2d view (b) 3D isometric view

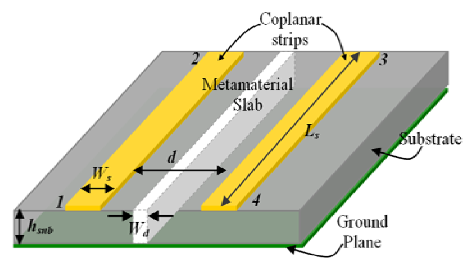

Fig. 2. A parallel coupled microstrip line separated by a DNG metamaterial slab.

mutual coupling between adjacent microstrip lines. A comparison between both computer simulation statistics for both formulations namely Mobius-FDTD algorithm and hybrid (Mobius-ADE) FDTD algorithm is also given. In Section-IV, a parallel version of the code is reported and much computational effort is discussed. This is followed by concluding remarks in Section V.

II. MATHEMATICAL FORMULATION OF MOBIUS-FDTD AND MOBIUS-ADE-FDTD ALGORITHM

The Maxwell's curl equations in Laplace domain (s-domain) normalized by Gaussian unit convention are given by:

$$s\vec{D}(\vec{r},s)=\frac{1}{\mu_0\varepsilon_0}(\nabla\times\vec{H}(\vec{r},s)) \quad (1)$$

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$$s\vec{B}(\vec{r},s)=-\frac{1}{\mu_0\varepsilon_0}(\nabla\times\vec{E}(\vec{r},s)) \tag{2}$$

The normalized s-domain constitutive relationships interconnecting the electric and magnetic field and flux densities in a general dispersive media are given by:

$$\vec{D}(\vec{r},s)=\varepsilon_r(s)\vec{E}(\vec{r},s) \tag{3}$$

$$\vec{B}(\vec{r},s)=\mu_r(s)\vec{H}(\vec{r},s) \tag{4}$$

Drude and Lorentz models respectively are considered for characterizing the dispersive permittivity and permeability functions of a homogeneous double negative (DNG) metamaterial media as illustrated in the equations below [11]:

$$\varepsilon_r(s)=\varepsilon_\infty+\frac{\omega_{pe}^2}{s(\nu_{ce}+s)} \tag{5}$$

$$\mu_r(s)=\mu_\infty+\frac{(\mu_s-\mu_\infty)\omega_{pm}^2}{\omega_{pm}^2+s\delta+s^2} \tag{6}$$

In Drude model Eq (5) for permittivity, ω_{pe} and ν_{ce} denote the electric plasma frequency (angular) and electric damping frequency respectively and ε_∞ is the high frequency limit of permittivity. In the Lorenz model Eq (6) for permeability, there is magnetic radial resonant angular frequency (ω_{pm}), magnetic damping frequency (δ) along with the high and low frequency limits for permeability (μ_∞ and μ_s respectively).

The analysis of the curl equations (1) and (2) can be done by using standard FDTD update equations [5], but formulation of suitable difference equations for calculation of fields ($\vec{E},\vec{H}$) from flux densities ($\vec{D},\vec{B}$) needs some efficient mathematical treatment. It is observed that $\mu_r(s)$ and $\varepsilon_r(s)$ can be treated as s-domain transfer functions, which are indeed invertible. So one can construct their respective inverse functions denoted by $G_\mu(s)=[\mu_r(s)]^{-1}$ and $G_\varepsilon(s)=[\varepsilon_r(s)]^{-1}$. Following this, z-domain equivalent transfer functions $G_\mu(z)$ and $G_\varepsilon(z)$ are obtained by applying standard z-transform or Mobius transform on the inverse functions $G_\mu(s)$ and $G_\varepsilon(s)$. The conversion from s-domain to z-domain is crucial because it allows us to utilize the digital filter theory on these transfer functions to obtain the corresponding time-domain equations. The Lorenzian permeability function as shown in (6) cannot be converted to z-domain using standard z-domain mapping ($\frac{1}{s}\rightarrow\frac{\Delta t}{1-z^{-1}}$), and so Auxiliary Differential Equation (ADE) method can be applied to get time domain update equations [12].

A. *Mobius-FDTD Method*

In this section the Mobius transform formulation of FDTD-update equations for both the constitutive relations (3) and (4) are given. It is well-known that Mobius transform is mathematically similar to the bilinear transform, where the well-known mapping equation [6] is given in (7).

$$s=\frac{2}{\Delta t}\frac{1-z^{-1}}{1+z^{-1}} \tag{7}$$

Hence the equations (3) and (4) can be rewritten in the z-domain as:

$$\vec{E}(\vec{r},z)=G_\varepsilon(z)\vec{D}(\vec{r},z) \tag{8}$$

$$\vec{H}(\vec{r},z)=G_\mu(z)\vec{B}(\vec{r},z) \tag{9}$$

where

$$G_\varepsilon(z)=\frac{b_{e0}+b_{e1}z^{-1}+b_{e2}z^{-2}}{1+a_{e1}z^{-1}+a_{e2}z^{-2}} \tag{10}$$

$$G_\mu(z)=\frac{b_{h0}+b_{h1}z^{-1}+b_{h2}z^{-2}}{1+a_{h1}z^{-1}+a_{h2}z^{-2}} \tag{11}$$

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The quantities $b_{ei}, b_{hi}, a_{ej}, a_{hj}$ (i=0, 1, 2; j=1, 2) along with new variables d_{e0}, d_{h0} can be calculated for the DNG medium parameters $\varepsilon_{\infty}, \omega_{pe}, \nu_{ce}, \mu_{\infty}, \mu_s, \omega_{pm}$ and δ using the equations below.

$$d_{e0}=\left(\frac{4\varepsilon_{\infty}}{\Delta t^2}+\frac{2\nu_{ce}}{\Delta t}+\frac{\omega_{pe}^2}{\varepsilon_0}\right) \quad (12)$$

$$b_{e0}=\frac{1}{d_{e0}}\left(\frac{2}{\Delta t}\left(\nu_{ce}+\frac{2}{\Delta t}\right)\right) \quad (13)$$

$$b_{e1}=\frac{1}{d_{e0}}\left(\frac{-4}{\Delta t}\right) \quad (14)$$

$$b_{e2}=\frac{1}{d_{e0}}\left(\frac{-2}{\Delta t}\left(\nu_{ce}-\frac{2}{\Delta t}\right)\right) \quad (15)$$

$$a_{e1}=\frac{1}{d_{e0}}\left(\frac{-8\varepsilon_{\infty}}{\Delta t^2}+\frac{2\omega_{pe}^2}{\varepsilon_0}\right) \quad (16)$$

$$a_{e2}=\frac{1}{d_{e0}}\left(\frac{4\varepsilon_{\infty}}{\Delta t^2}-\frac{2\nu_{ce}}{\Delta t}+\frac{\omega_{pe}^2}{\varepsilon_0}\right) \quad (17)$$

$$d_{h0}=\mu_s\omega_{pm}^2+\frac{2\mu_{\infty}\delta}{\Delta t}+\frac{4\mu_{\infty}}{\Delta t^2} \quad (18)$$

$$b_{h0}=\frac{1}{d_{h0}}\left(\omega_{pm}^2+\frac{2\delta}{\Delta t}+\frac{4}{\Delta t^2}\right) \quad (19)$$

$$b_{h1}=\frac{2}{d_{h0}}\left(\omega_{pm}^2-\frac{4}{\Delta t^2}\right) \quad (20)$$

$$b_{h0}=\frac{1}{d_{h0}}\left(\omega_{pm}^2-\frac{2\delta}{\Delta t}+\frac{4}{\Delta t^2}\right) \quad (21)$$

$$a_{h1}=\frac{2}{d_{h0}}\left(\mu_s\omega_{pm}^2-\frac{4\mu_{\infty}}{\Delta t^2}\right) \quad (22)$$

$$a_{h2}=\frac{1}{d_{h0}}\left(\mu_s\omega_{pm}^2-\frac{2\mu_{\infty}\delta}{\Delta t}+\frac{4\mu_{\infty}}{\Delta t^2}\right) \quad (23)$$

Now (10) and (11) can be interpreted as infinite-impulse response digital filters which can be implemented by using the transposed direct form as given in [13]. New fictitious vectors $\vec{W}_{e1}, \vec{W}_{e2}, \vec{W}_{h1}$ and $\vec{W}_{h2}$ are introduced for this purpose. The resulting time-update equations for $\vec{E}^{n+\frac{1}{2}}$ and $\vec{H}^{n+1}$ are shown in the following equations.

$$\vec{E}^{n+\frac{1}{2}}=b_{e0}.\vec{D}^{n+\frac{1}{2}}+\vec{W}_{e1}^{n-\frac{1}{2}} \quad (24)$$

$$\vec{W}_{e1}^{n+\frac{1}{2}}=b_{e1}.\vec{D}^{n+\frac{1}{2}}-a_{e1}.\vec{E}^{n+\frac{1}{2}}+\vec{W}_{e2}^{n-\frac{1}{2}} \quad (25)$$

$$\vec{W}_{e2}^{n+\frac{1}{2}}=b_{e2}.\vec{D}^{n+\frac{1}{2}}-a_{e2}.\vec{E}^{n+\frac{1}{2}} \quad (26)$$

$$\vec{H}^{n+1}=b_{h0}.\vec{B}^{n+1}+\vec{W}_{h1}^{n} \quad (27)$$

$$\vec{W}_{h1}^{n+1}=b_{h1}.\vec{B}^{n+1}-a_{h1}.\vec{H}^{n+1}+\vec{W}_{h2}^{n} \quad (28)$$

$$\vec{W}_{h2}^{n+1}=b_{h2}.\vec{B}^{n+1}-a_{h2}.\vec{H}^{n+1} \quad (29)$$

Since a particular problem space comprising of homogeneous DNG MTM media as well as standard double positive (DPS) materials like dielectrics, conductors and free-space is to be analyzed, so it is desirable to compute the values of the parameters $b_{ei}, b_{hi}, a_{ej}, a_{hj}$ (i=0,1,2; j=1,2) for DPS medium also. In this way one can treat equations (24) to (29) as generalized update equations, compatible for both DNG and DPS media. It is known that standard lossy dielectrics can be considered that has the relative permittivity and permeability functions of the form.

$$\varepsilon_r(s)=\varepsilon_r+\frac{\sigma}{s\varepsilon_0} \quad (30)$$

$$\mu_r(s)=1 \quad (31)$$

For free space we have $s=0, e_r=1$. For simulation of conducting patches we make the tangential component of electric field zero by making $\sigma_m \rightarrow \infty$ (where m=x, y or z) at the necessary planes. Following similar procedure as for dispersive metamaterials, we get the following parameter-expressions for characterizing DPS media in Mobius formulation scheme.

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$$b_{e0}=\frac{2}{2\varepsilon_r+\frac{\sigma\Delta t}{\varepsilon_0}} \quad (32)$$

$$b_{e1}=-b_{e0} \quad (33)$$

$$a_{e1}=\frac{\frac{\sigma\Delta t}{\varepsilon_0}-2\varepsilon_r}{\frac{\sigma\Delta t}{\varepsilon_0}+2\varepsilon_r} \quad (34)$$

$$b_{h0}=1 \quad (35)$$

$$b_{e2}=a_{e2}=b_{h1}=b_{h2}=a_{h1}=a_{h2}=0 \quad (36)$$

B. Mobius-ADE-FDTD Method

Next the hybrid algorithm that deals with Drude model Eq (5) using Mobius transform method and Lorentz model Eq (6) using ADE method is discussed here. The ADE formulation technique (introducing a fictitious vector-quantity $\vec{Sm}$) as explained in [12] is used for obtaining $\vec{H}^{n+1}$ from $\vec{B}^{n+1}$. The final update equations and new parameter-set (C_1,C_2,C_3) are shown in (37) to (41).

$$C_{h1}=\frac{(\mu_s-\mu_\infty)\omega_{pm}^2\Delta t^2}{0.5\delta\Delta t+1} \quad (37)$$

$$C_{h2}=-\frac{2-\omega_{pm}^2\Delta t^2}{0.5\delta\Delta t+1} \quad (38)$$

$$C_{h3}=\frac{0.5\delta\Delta t-1}{0.5\delta\Delta t+1} \quad (39)$$

$$\vec{Sm}^{n+1}=C_{h1}.\vec{H}^n+C_{h2}.\vec{Sm}^n+C_{h3}.\vec{Sm}^{n-1} \quad (40)$$

$$\vec{H}^{n+1}=\frac{1}{\mu_\infty}(\vec{B}^{n+1}-\vec{Sm}^{n+1}) \quad (41)$$

Equations (24) to (26) and corresponding parameters (b_{e0},b_{e1},b_{e2},a_{e1} and a_{e1}) calculate and update values of $\vec{E}^{n+\frac{1}{2}}$ from $\vec{D}^{n+\frac{1}{2}}$ as in the complete Mobius-FDTD method.

III. RESULTS AND DISCUSSION

The dispersive FDTD formulation for metamaterial media was tested for parallel coupled microstrip lines separated by a metamaterial slab as indicated in Fig.2. Parallel coupled microstrip lines are indispensable elements in many microwave circuits like couplers and filters [14, 15] and also in printed circuit boards (PCBs). The coupling between two identical parallel microstrip lines mounted on the same dielectric substrate is dependent of their relative spacing that has to be pre-specified for any problem of compact system design. The degree of forward mutual coupling between two microstrip lines as shown in Fig.2 can be quantitatively determined by the scattering parameter S_{31}(dB), since S_{31} is the measure of the power coupled to port-3 (receiving port for adjacent line) from port-1 (transmitting port for main line). In general, when the relative line spacing is fixed, S_{31}(dB) increases as the frequency of operation is increased. This can yield serious problems of cross-talk if the parallel lines are used as independent information-channels of signals at microwave frequencies. Analysis shows that a DNG medium slab of suitable parameter specifications can be placed in between the adjacent lines for crosstalk elimination at a particular frequency.

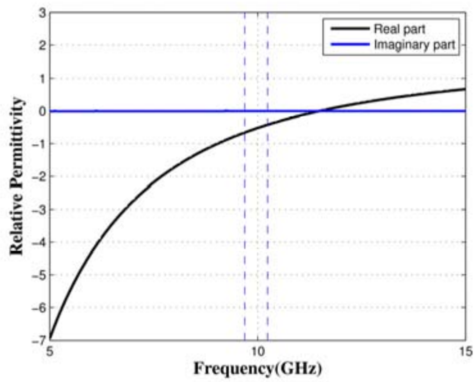

Fig. 3. Variation of real and imaginary parts of ε_r with frequency.

In the coupled microstrip line system as shown in Fig. 2, each line having length L_s of 36 mm, width W_s of 2.334 mm and characteristic impedance of 50Ω. The spacing between the lines is d=3.89 mm. The substrate used is of height h_{sub} as 0.795 mm and dielectric constant ε_{sub}=2.2. The ports 1, 2, 3 and 4 are numbered accordingly as illustrated in Fig.2. A DNG metamaterial slab having height and length same as the substrate and width 1.556 mm is sandwiched between the two adjacent microstrip lines. The metamaterial slab is considered to possess material properties specified by the equivalent extracted parameters in [11]. The metamaterial slab is characterized by $\omega_{pe}=2\pi\times14.63$ GHz, ν_{ce}=30.69 MHz, $\omega_{pm}=2\pi\times9.67$ GHz, δ=1.24 GHz, ε_{∞}=1.62, μ_s=1.26 and μ_{∞}=1.12. With the given specifications, the value of relative permittivity remains negative over the frequency range 5-11 GHz as illustrated in Fig.3, whereas the permeability is negative for a smaller range 9.69-10.24 GHz as shown in Fig.4. So the slab acts as a proper DNG medium within that narrow range, as enclosed by dashed lines.

The problem space is analyzed first by Mobius-FDTD and then the hybrid Mobius-ADE-FDTD method. The grid sizes taken are Δx=0.389 mm, Δy=0.4 mm and Δz=0.265 mm. The sampling time Δt is 0.441 ps. After discretizing the geometry a first derivative Gaussian pulse is launched in port one to extract time domain information. The time-domain samples of the z-component of the electric field are calculated at all the port terminal planes. The scattering matrices S_{11}, S_{21}, S_{31} and S_{41} in dB are computed using Fourier transform of the time domain samples. The variation of S_{11} and S_{21}, not shown here, indicate that the input port 1 is matched and that forward transmission to port 2 is possible without losses. It is seen from the plots of S_{31}(dB) versus frequency (GHz), as given in Fig 5, the mutual coupling without the DNG slabs exhibit a smooth variation. A sharp

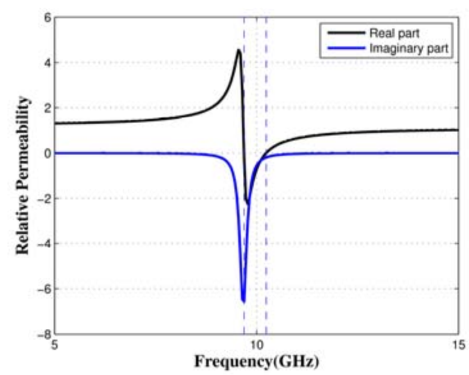

Fig. 4. Variation of real and imaginary parts of μ_r with frequency.

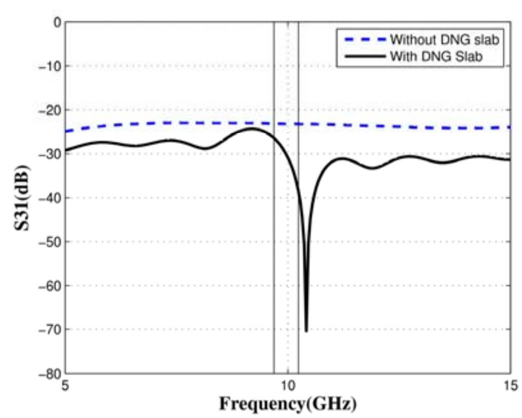

Fig. 5. Variation of forward mutual coupling with frequency; A dip in the frequency range where the slab acts as perfect DNG slab is observed.

reduction of mutual coupling just above the frequency range where the metamaterial slab behaves as perfect DNG media is observed. In equivalent circuit theoretic terms it can be interpreted that resonant notch filter-type characteristics have been incorporated in the coupled microstripline system by introduction of DNG slab. The DNG material parameters can be tuned for manipulating the cross-talk rejection frequency.

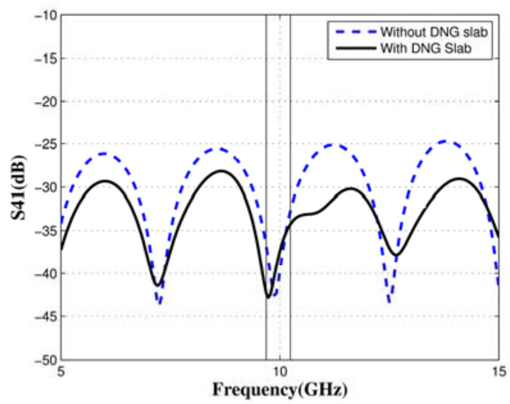

Fig.6. Variation of backward coupling with frequency.

From full-wave analysis it is observed that there is no drastic change in the backward coupling S_{41}(dB) when DNG medium is used, although the levels of coupling gets reduced by 3-5 dB at the characteristic frequencies as indicated in Fig.6. This phenomenon is mainly due to the absorption induced by the DNG metamaterial slab. Finally it is to be noted that simulation-results for pure Mobius-FDTD codes and hybrid FDTD codes come out to be the same, although the time taken for execution is lesser for the pure Mobius-FDTD code by 20 secs. The reason behind this is presence of fewer numbers of intermediate matrices that are updated during the execution of main FDTD iterative loop.

IV. NOTE ON COMPUTATIONAL EFFORT

The serial FDTD code, with the algorithm as prescribed in [5] was implemented using MATLAB™ for analysis purpose. For the FDTD mesh size pertaining to the problem formulation, as illustrated in Fig. 2, took a computational time (starting from initialization of different parameter values to storage of calculated time-domain E and H-field data for further computation in frequency domain) of 4000 seconds. The large computational time was mainly due to the fact that the pseudo-code description of the FDTD algorithm in [5] involves the sequential repetition of nested for loops, especially during the calculation of field matrices.

To overcome this difficulty, a vectorized version of the code was developed which is much concise and utilizes the advantage of MATLAB™ optimization for vector and matrix operations, hence ensures faster execution. The same code was run in machines of different processor configurations and the run-time was observed. It was seen that Pentium-IV processor took 2500 secs (approx.) of run-time, whereas dual-core and quad-core processors exhibited considerable speed-up with simulation time of 1500 secs and 845 secs respectively.

V. CONCLUSIONS

In this work Mobius-transform technique and a hybrid Mobius-ADE FDTD algorithm to model electromagnetic wave propagation in metamaterial is presented. The negative effective permittivity and negative effective permeability is modeled as Drude and Lorentz medium respectively. The formulation is tested by full wave analysis of a metamaterial embedded parallel coupled microstrip line. The effect on coupling are studied which reveals reduction in forward coupling over the range where the slab behaves as a Double Negative (DNG) media. A parallel version of the full wave FDTD formulation for metamaterial is also developed which exhibits reduced run-time.

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REFERENCES

[1] D. R. Smith, W. J. Padilla, D. C. Vier, S. C Nemat-Nasser and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Phys. Rev. Lett.*, vol.84, no.18, pp.4184-4187., May 2000.

[2] J. B. Pendry, A. J. Holden, D. J. Robins and W. J. Stewart, "Magnetism from conductors and enhanced non-linear phenomena," *IEEE Transactions on Microwave Theory and Technique*, vol47, no.11, pp.2075-2084, Nov. 1999.

[3] C Caloz, A sanada and T Itoh, "A novel composite right/left handed coupled line directional coupler with arbitrary coupling level and broad bandwidth," *IEEE Trans. Microwave Theory and Techniques*, vol52, no.3, pp.980-992, March 2004.

[4] Lin, C. Caloz, and T. Itoh, "A branch-line coupler with two arbitrary operating frequencies using left-handed transmission lines," *IEEE-MTT Int. Symp. Dig., Philadelphia, PA*, vol. 1, pp. 325â"327, 2003.

[5] Denis M. Sullivan, "Electromagnetic Simulation using the FDTD Method", *IEEE Press Series on RF and Microwave Technology*, 2000.

[6] José A. Pereda, Ana Grande, Oscar González, and Ángel Vegas, "FDTD Modeling of Chiral Media by Using the Mobius Transformation Technique," *IEEE Antennas and Wireless Propagation Letters*, Vol. 5, pp.327-330, 2006.

[7] Jung-Yub Lee, Jeong-Hae Lee, Hyeong-Seok Kim, No-Weon Kang, and Hyun-Kyo Jung, "Effective Medium Approach of Left-Handed Material Using a Dispersive FDTD Method," *IEEE Transactions on Magnetics*, vol. 41, no. 5, pp.1484-1487, May 2005.

[8] Hossein Mosallaei, "FDTD-PLRC Technique for Modeling of Anisotropic-Dispersive Media and Metamaterial Devices," *IEEE Transactions on Electromagnetic Compatibility*, vol. 49, no. 3, pp.649-660, August 2007.

[9] Y.-Q. Zhang *y* and D.-B. Ge, "A Uunified FDTD Aapproach for Electromagnatic Analysis of Dispersive Object," *Progress In Electromagnetics Research,* PIER 96, pp.155-172, 2009.

[10] Riaz Siushansian and Joe Lo Vetri, "A comparison of Numerical Techniques for Modeling Electromagnetic Dispersive Media," *IEEE Microwave and Guided Wave Letters*, vol.5, no.12, pp.425-428, Dec 1995.

[11] G. Lubkowski, R. Schuhmann, and T. Weiland, "Extraction of effective metamaterial parameters by parameter fitting of dispersive models", *Microwave and Optical Technology Letters*, vol. 49, no. 2, pp. 285 February 2007.

[12] D Sarkar, S Sahu, R Ghatak, R K Mishra, D R Poddar, "FDTD Analysis of Coupled Microstrip Lines Separated by a DNG Slab", *Loughborough Antenna and Propagation Conference (IEEE)*, UK, 2010.

[13] J.G. Proakis and D.G. Manolakis, *Digital Signal Processing: Principles, Algorithms and Applications*, New York: Macmillan, 1992.

[14] D M Pozar, *Microwave Engineering*, John Wiley, 2nd Edition, 2004.

[15] Stephen C. Thierauf, *High-Speed Circuit Board Signal Integrity*, Boston, Artech House, Inc., 2004.