Circuit Synthesis for Compact Waveguide Filters with Closely-Spaced Frequency Selective Surfaces

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Abstract- This paper proposes a compact waveguide filter, which consists of resonators (called iris or FSS) inserted into a waveguide as transverse section at intervals of the length shorter than quarter wavelength. Such a filter is designed by an equivalent circuit approach. In this approach, we first derive resonant conditions of the filter, and also show the equivalent circuit to design the passband. Note that the resonators of compact filters are required to resonate at the frequency below the center frequency in the passband. Nevertheless, the filter can provide successfully the same passband as a conventional filter with quarter-wavelength inverters. The validity of the design approach is proven by the numerical results of a designed size-reduced filter using the method of moments.

Index Terms- Waveguide filters, frequency selective surfaces, resonance, size reduction.

I. INTRODUCTION

In recent years, resonant irises have been used as resonators for compact and lightweight waveguide filters [1–4], instead of relatively large cavity resonators. This type of filter consists of thin resonant irises (also called frequency selective surfaces (FSS) [5]) as resonators and quarter-wavelength $(\lambda_{g0}/4)$ waveguide sections as inverters. As a typical example, the filter in [1] arranges successively aperture iris resonators. To improve out-of-band characteristics, the length of inverter sections has been made shorter by loading inductive elements into the waveguide [2]. However, such a filter essentially needs an additional structure like the inductive element. On the other hand, we have already developed a high-performance bandpass filter introducing arbitrarily-shaped FSSs as resonators, thereby realizing multiple attenuation poles in stopbands at both sides of passband without any additional structures [5]. Therefore, if waveguide sections working as an inverter can be much shorter than $\lambda_{g0}/4$, such filters may become more compact and practical.

So this paper proposes an equivalent circuit approach for realizing very compact waveguide filters with iris or FSS as resonators located at very short interval. We first derive the resonant conditions from a dispersion equation for the whole filter structure and also the admittance inverter parameters for compact filters. As a result, it is shown that the resonators of compact filters are required to have the resonant frequency lower than the center frequency in the passband. As an example, the third-order waveguide filter is demonstrated at X band. The validity of the present approach is proven by the calculated results of a drastic size-reduced waveguide filter using the method of moments.

II. COMPACT FILTERS

A. Resonant Conditions

Figure 1(a) shows an example of a waveguide filter consisting of resonators which are located into a waveguide at intervals of the length *l*. To make the discussion simple, we consider a third-order filter shown in Fig. 1(a), and neglect higher-order mode interaction between resonators. In this paper, the resonators are expressed as the shunt susceptance $Y_i = jB_i$ (i = 1, 2, 3), and also the transmission lines between them has the electrical length ϕ as shown in Fig. 1(b).

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Fig. 1. Waveguide filter consisting of thin resonators (called iris or FSS). (a) Filter structure and (b) its equivalent circuit network.

To realize a compact waveguide filter, resonant conditions are derived here using a dispersion equation for the infinite periodic structure, of which the unit cell is the equivalent circuit network in Fig. 1(b) that has total physical length p = 2l. Then, the dispersion equation is as follows [6]:

$$\beta = \frac{1}{p} \cos^{-1} \left(\frac{1}{2} (A + D) \right), \tag{1}$$

where β is the propagation constant, p (= 2l) is the periodic spacing corresponding to the length of the unit cell. A and D denote ABCD matrix elements of the unit cell. Figure 2 shows a typical dispersion characteristic for the one-dimensional periodic symmetric-filter structure, where we set $Y_1 = Y_3$. The similar dispersion characteristic has been already reported in [7]. The dispersion curve has both the right-handed (RH) and the left-handed (LH) passbands, and also a very bandgap narrow appears between them. Therefore, there exist four cutoff angular frequencies at $\omega = \omega_1, \omega_2, \omega_3, \omega_4$ in this curve. The cutoff angular frequencies $\omega = \omega_1$, ω_4 and $\omega = \omega_2, \, \omega_3 \, (\approx \omega_0)$, where ω_0 is the center angular frequency of the passband, can be obtained by



Fig. 2. Dispersion characteristic for one-dimensional infinite periodic structure having the third-order symmetric filter shown in Fig. 1(b) as a unit cell.



Fig. 3. Resonant circuits at the angular frequency ω_2 or ω_3 very close to the center angular frequency ω_0 in passband.

substituting $\beta = 0$ and $\beta = \pi/p$ into (1), respectively. As a result, we can get the following equations:

$$\begin{pmatrix} 1 - \frac{B_1 B_2}{2} \end{pmatrix} + \begin{pmatrix} B_1 + \frac{B_2}{2} \end{pmatrix} Y_0 \cot \phi = 0$$

$$at \ \omega = \omega_1, \omega_4,$$

$$\begin{pmatrix} B_1 - Y_0 \cot \phi = 0 \\ B_1 - Y_0 \cot \phi = 0 \end{pmatrix}$$

$$(2)$$

$$\begin{cases} \frac{B_1}{2} - Y_0 \cot \phi = 0 \\ \frac{B_2}{2} - Y_0 \cot \phi = 0 \end{cases} \text{ at } \omega = \omega_2, \omega_3, \qquad (3)$$

where Y_0 represents the characteristic admittance of the line section. The former equation determines the bandwidth of the passband for the infinite periodic structure. The bandgap appears only for the infinite periodic structure, and yet disappears by setting $B_1 = B_2/2$. The important equation for realizing compact filters is (3) that represents the resonant conditions near the center frequency. The resonant conditions in (3) suggest that equivalent circuits displayed in Fig. 3 resonate at the angular frequency ω_2 or ω_3 very

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Fig. 4. Resonators and inverters for compact filter design of the third-order waveguide filter.



Fig. 5. Equivalent network expression using J inverters obtained from Fig. 4.

close to ω_0 . In the case of quarter-wavelength inverters, the electrical length ϕ has $\pi/2$ radians at ω_0 , that is, $\cot \phi = 0$. Then, the resonant condition in (3) leads to $B_1 = 0$ and $B_2 = 0$. These equations mean that the resonators are designed to resonate at the center frequency. In the case of $\cot \phi \neq 0$, namely, closely-spaced resonators, the resonators are required to satisfy the condition in (3), instead of the resonance at the center frequency, that is $B_1 = B_2 = 0$.

B. Inverter Expression

Let us consider an equivalent circuit expression using admittance inverters (*J* inverters) to design a passband required for waveguide filters. From the resonant conditions in (3), the shunt susceptance $-Y_c = -jY_0 \cot \phi$ is loaded parallel to the original resonators Y_i (i = 1, 2, 3) as shown in Fig. 4. It can be found from (3) that the equivalent resonators (Y_1-Y_c) and (Y_2-2Y_c) resonate at the center frequency f_0 . In addition, the shunt susceptance Y_c is loaded on each side of the transmission line so that the circuit in Fig. 4 is equivalent to the original circuit shown in Fig. 1 (b). Then, the *ABCD* matrix of the network surrounded by the dashed line regarded as inverter in Fig. 4 is obtained as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & j\sin\phi \\ j\frac{1}{\sin\phi} & 0 \end{bmatrix},$$
 (4)

where we set $Y_0 = 1$ for normalization. The matrix indicates that the network works as an ideal *J* inverter having the parameter $J = 1/\sin\phi$. Hence, the compact waveguide filter can be expressed as the equivalent filter network shown in Fig. 5, where the *J* inverters connected to the input/output waveguides have $J_{01} = J_{34} = 1$, and the *J* inverters between the resonators have J_{12} $= J_{23} = 1/\sin\phi$. The resonators Y_{ri} in Fig. 5 can be also represented in the following forms:

$$Y_{r1} = j\omega C_{r1} + \frac{1}{j\omega L_{r1}}$$
(5)
$$= j\omega C_{1} + \frac{1}{j\omega L_{1}} - jY_{0} \cot\phi,$$
(7)
$$Y_{r2} = j\omega C_{r2} + \frac{1}{j\omega L_{r2}}$$
(6)
$$= j\omega C_{2} + \frac{1}{j\omega L_{2}} - j2Y_{0} \cot\phi,$$
(7)
$$= j\omega C_{3} + \frac{1}{j\omega L_{3}} - jY_{0} \cot\phi,$$
(7)

where L_{ri} and C_{ri} (i = 1, 2, 3) denote the parameters of parallel resonant circuits Y_{ri} obtained from a conventional inverter approach [8], L_i and C_i (i = 1, 2, 3) denote those of the resonators Y_i in Fig. 1(b). These equations support only the characteristic at around the passband because they include the frequency-dependent parameter of the electrical length ϕ . Note that $L_{ri}C_{ri} = 1/\omega_0^2$ for the equivalent network representation in Fig. 5, but $L_i C_i \neq 1/\omega_0^2$ for original resonators the (admittance Y_i) in Fig. 1(b).

C. Passband Design

Using the above results, a passband for compact waveguide filters can be designed. First, the physical length l between resonators and the passband specifications are given. Then the parameters L_{ri} and C_{ri} for the equivalent resonators Y_{ri} in Fig. 5 can be easily obtained from a conventional filter design [8] using J_{01}

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Fig. 6. Comparison of frequency characteristics calculated by equivalent circuit approach between compact filter with l = 5.0 [mm] ($\approx \lambda_{g0}/8$) and conventional one with $l = \lambda_{g0}/4$. *LC* parameters for l = 5.0 [mm]: $L_1 = L_3 = 1.168$ [pH], $C_1 = C_3 = 0.232$ [nF], $L_2 = 0.362$ [pH], $C_2 = 0.729$ [nF].



Fig. 7. Transmission response of each resonator for compact filter with l = 5.0 [mm] calculated by equivalent circuit approach.

= $J_{34} = 1$ and $J_{12} = J_{23} = 1/\sin\phi_0$, where ϕ_0 represents the electrical length at the center frequency f_0 . Next, the parameters L_i and C_i of the original resonators Y_i in Fig. 1(b) is determined to satisfy the relations in Eqs. (5), (6) and (7) within the specified passband. After the above procedure, a little adjustments of L_i and C_i are required because actually J inverter parameter ($1/\sin\phi$) depends on the frequency. When the frequency response is satisfied with the specification, the passband design using the equivalent circuit is finished. The shape of the resonators (called iris or FSS) can be easily designed by using the method based on both the present equivalent circuit approach and a genetic algorithm (GA) [9] as described in [5]. If the FSSs developed by us are used, the filter can also produce multiple attenuation poles without any additional structures.

III. DESIGN EXAMPLE

As an example, we design a compact waveguide filter, of which the passband response is approximated by 3-pole 0.01dB Chebyshev response at the center frequency $f_0 = 10$ [GHz] and the bandwidth $f_w = 400$ [MHz]. The main rectangular waveguide is WR-90 (a = 22.86[mm], b = 10.16 [mm]). Here, the length of the interval between resonators is chosen to be 5.0 mm that is about $\lambda_{g0}/8$. Figure 6 shows the comparison of frequency characteristics between a compact filter with l = 5.0 [mm] and a $l = \lambda_{g0}/4$. conventional one with These characteristics are calculated by the equivalent circuit approach. It can be found from this figure that the compact filter can produce the same passband response as the conventional one. The transmission response required for each resonator Y_i in Fig. 1(b) is shown in Fig. 7. We can observe that the resonators 1 and 3, and 2 have the resonant frequency f_{r1} and f_{r2} below the center frequency f_0 , respectively. Although any resonators do not resonate at the center frequency f_0 as shown here, the compact filter can provide successfully the specified passband by satisfying the resonant conditions in (3).

Finally, we present a compact waveguide filter using FSSs designed by the GA to realize four attenuation pole frequencies under the same passband specification [5]. The designed FSS geometries are displayed in Fig. 8. Each FSS is supported by the dielectric substrate with the relative permittivity $\varepsilon_r = 3.8$ and the thickness h= 0.4 [mm]. The FSSs are designed to resonate at the frequency f_{r1} and f_{r2} , as shown in Fig. 9. Figure 10 shows the frequency characteristics of the designed filter calculated by the cascade connection of TE₁₀-mode scattering matrix obtained from the method of moments. The length *l* between the FSS resonators is 4.6 mm

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Fig. 8. GA-designed FSS geometry for compact waveguide filter (black parts represent conductors).



Fig. 9. Transmission responses of GA-designed FSSs calculated by the method of moments (heavy lines) in comparison with those of the equivalent circuit approach (thin lines).

for constructing the passband in consideration of the dielectric effect. The designed filter can realize the drastic size reduction, and can provide the four attenuation poles, with keeping the passband response. The above results validate the discussion of the equivalent circuit approach for compact filters in the previous section.

IV. CONCLUSION

This paper has developed a compact waveguide filter, which consists of the resonators (called iris or FSS) inserted into a waveguide as transverse sections at intervals of the length shorter than quarter wavelength. We have derived here the resonant conditions of such compact filters, and have obtained the equivalent circuit expression to design the specified passband. The validity of the approach for compact filter design has been proven by the numerical analysis using the method of moments for the designed filter.

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