Systematic Extraction Method for the Determination of HBT Temperature-Dependent DC Model Parameters

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Abstract- This paper presents a systematic parameter extraction method for an HBT temperature-dependant equivalent-circuit DC model. A reliable procedure was developed for determining the HBT thermal resistance, requiring only forward Gummel data at different temperatures and collector-emitter bias voltages. The extraction method was applied to predict the DC characteristics of a 2x25 μm² emitter-area InGaP/GaAs HBT device.

Index Terms- Heterojunction bipolar transistor, DC equivalent-circuit model, self-heating, parameter extraction.

I. INTRODUCTION

Heterojunction bipolar transistors (HBT’s) have been widely accepted in a variety of microwave applications such as power amplifiers, oscillators, and mixers [1]. A key issue in HBT modeling is the inclusion of self-heating. This effect manifests its presence in the behavior of the current-voltage ($I_c-V_{ce}$) characteristics, when the transistor is operated at high power densities. For a reliable design, a careful determination of the thermal resistance is required. Different methods have been proposed [2]-[4]. Of particular interest is the one developed by Bovolon et al. [4]. This method is simple and requires only the measurement of the device DC output characteristics at two different temperatures.

In this paper, an HBT temperature-dependant DC model is presented. Self-heating is accounted for through the variation with temperature of the base-emitter built-in voltage. A new method for the thermal resistance determination is presented. All diodes parameters, including saturation currents, ideality factors, and self-heating coefficients, are directly extracted from forward and reverse Gummel measurements at different ambient temperatures and dissipated powers. The developed HBT DC model was implemented in ADS as an SDD (Symbolically Defined Device) and validated using measured DC characteristics of a 2x25 μm² emitter-area InGaP/GaAs HBT.

II. MODEL DESCRIPTION

A. Model Parameters

The adopted HBT large-signal equivalent-circuit model is shown in Fig. 1. Also included in this figure is a thermal equivalent circuit, which is used to account for temperature rise at the base-emitter junction. Parasitic capacitances and inductances are not shown in figure 1 as they are de-embedded from S-parameters measurements using the technique published in [5]. $R_b$, $R_c$ and $R_e$ are the series resistances and they are considered as bias-independent. $R_{bb}$ is the intrinsic base resistance. It is a bias-dependent parameter. All resistances are extracted from S-parameter measurements [5]. $C_{be}$ is the base-emitter capacitance. $C_{bc}$ and $C_{ce}$ are the extrinsic...
and intrinsic base-collector capacitances, respectively. In this paper, only in the DC part of the large-signal model is considered.

Diodes $D_{bf3}$ and $D_{br}$ represent the currents flowing through the base-emitter and base-collector junctions, respectively. $D_{bf1}$ model the reverse hole-injection from base to emitter while $D_{bf2}$ model the recombination effects in the base-emitter space charge region and at the emitter surface. It is assumed here that all currents represented by diodes obey the Shockley relation:

$$I = I_s (\exp \left(\frac{V}{N V_T}\right) - 1)$$  \hspace{1cm} (1)

where $I_s$ and $N$ are respectively the saturation current and ideality factor, and $V_T$ is the thermal voltage (= $kT/q$) at the junction temperature $T$.

The collector-emitter current source $I_{cT}$ in the model is calculated from:

$$I_{cT} = I_{sf} - I_{sr}$$  \hspace{1cm} (2)

where $I_{sf}$ and $I_{sr}$ are respectively the forward and reverse current components.

The total forward base current is the sum of the current components $I_{bf1}$, $I_{bf2}$ and $I_{bf3}$ corresponding respectively to diodes $D_{bf1}$, $D_{bf2}$ and $D_{bf3}$. The reverse base-collector current component $I_{br}$ corresponds to diode $D_{br}$.

### B. Thermal Effects

It is assumed that the saturation current and ideality factor remain fixed at their ambient temperature values while the current dependence on junction temperature is attributed to an increase in the built-in voltage of the base-emitter junction by an incremental thermal voltage $V_{TH}$ as in [6], where it was assumed that:

$$V_{TH} = \frac{\partial V_{be}}{\partial T} \bigg|_{I_e=const} \Delta T$$

In the present paper, this equality condition is relaxed by assuming only proportionality between the left- and right-hand-side terms i.e.

$$V_{TH} = \alpha_e \frac{\partial V_{be}}{\partial T} \bigg|_{I_e=const} \Delta T$$  \hspace{1cm} (3)

$\alpha_e$ is a proportionality factor. The introduction of this factor allows assessing the validity of the equality hypothesis advanced by [6] by observing how far from unity is the value of $\alpha_e$.

### III. PARAMETER EXTRACTION PROCEDURE

#### A. Extraction of the Parameter $\partial V_{be}/\partial T|_{I_e}$

The input power is assumed to be dissipated at the base-emitter junction and it is given by:
\[ P_{d_{sp}} = I_c V_{ce} + I_b V_{be} = I_c V_{ce} \]  

(4)

Using forward Gummel measurements (Fig. 2), \( V_{be} \) is determined as a function of the ambient temperature, at constant \( I_E \) and \( V_{ce} \) (or equivalently, at constant \( I_E \) and \( P_{d_{sp}} \) As \( I_E \approx I_c \)). This dependence is linear in the temperature range of interest, as shown in Fig. 3, and a single coefficient \( \partial V_{be} / \partial T \) can be determined for each value of \( P_{d_{sp}} \) (Fig. 4).

**B. Extraction of the Thermal Resistance**

The proposed procedure for determining the thermal resistance uses forward Gummel measurements at different ambient temperatures and different bias voltages \( V_{ce} \). This procedure extends the method developed by Dawson et al. [7]. Indeed, as proven in [7], at constant emitter current \( I_E \), the base-emitter voltage \( V_{be} \) varies linearly with the junction temperature \( T_j \). Thus, around an arbitrary temperature \( \theta \) (\( \theta \geq T_0 \), \( T_0 \) being the substrate temperature), the voltage \( V_{be} \) at temperature \( T_j \) can be written as follows:

![Fig. 2. Forward Gummel measurements at different ambient temperatures](image1)

![Fig. 3. Base-emitter voltage \( V_{be} \) measurements at different temperatures and emitter currents \( I_E \)](image2)

![Fig. 4. Variation of the term \( \partial V_{be} / \partial T \) versus dissipated power](image3)
Knowing that $T_j = T_0 + R_{th} P_{dsp}$ (by definition of the thermal resistance), the relation (5) becomes:

$$V_{be}(T_j, P_{dsp}) = V_{be}(0) + \frac{\Delta V_{be}}{\Delta T}
(T_0 - 0) + \frac{\Delta V_{be}}{\Delta T} R_{th} P_{dsp}$$

(6)

If $P_{dsp}$ is kept constant, while the substrate temperature is changed from $T_{01}$ to $T_{02}$, then the coefficient $\Delta V_{be} / \Delta T |_{0}$ can be determined. On the other hand, if the substrate temperature is kept constant while the dissipated power is changed from $P_{dsp1}$ to $P_{dsp2}$, then, using the expression for $\Delta V_{be} / \Delta T |_{0}$, the thermal resistance is given by:

$$R_{th}(P_{dsp}) = \frac{V_{be}(T_{01}, P_{dsp1}) - V_{be}(T_{02}, P_{dsp2})}{V_{be}(T_{01}, P_{dsp1}) - V_{be}(T_{02}, P_{dsp2})}$$

(7)

Then, if we repeat this procedure for different values of the emitter current $I_E$, we can calculate the variation of the thermal resistance versus the dissipated power, as illustrated in Fig. 5.

C. Determination of Forward Collector Current Parameters

The forward collector current is defined by:

$$I_{cf} = I_{cf}(\exp\left(\frac{V_{heint} + \alpha_1 \frac{\partial V_{be}}{\partial T}}{n_{cf} V_0}\right) - 1)$$

(8)

$$V_{heint} = V_{be} - R_c (I_{cf} + I_{cf}) - R_b I_{bf}$$

(9)

$V_{heint}$ and $V_{be}$ are respectively the internal and external base-emitter voltages. $V_0$ is the thermal voltage at the ambient temperature $T_0$.

The collector current expression (8) becomes:

$$\log(I_{cf}) - \log(I_{cf}) = \log(\exp\left(\frac{V_{heint} + \alpha_1 \frac{\partial V_{be}}{\partial T}}{n_{cf} V_0}\right) - 1)$$

(10)

Taking an initial bias-point ($I_{cfi}$, $V_{bei}$) from the forward Gummel measurements and using relation (10), one can eliminate the term including the saturation current ($\log(I_{scf})$) as follows:

$$\log(I_{cf}) - \log(I_{cf}) = \log(\exp\left(\frac{V_{heint} + \alpha_1 \frac{\partial V_{be}}{\partial T}}{n_{cf} V_0}\right) - 1) - \log(\exp\left(\frac{V_{heint} + \alpha_1 \frac{\partial V_{be}}{\partial T}}{n_{cf} V_0}\right) - 1)$$

(11)

with $I_{cf} > I_{cfi}$ and $V_{be} > V_{hei}$.

Fig.5. Thermal resistance versus dissipated power
Thus for each bias-point \((I_{cfj}, V_{bej})\) from forward Gummel measurements, one can define a local error function \(FCT_{ij}\) as follows:

\[
FCT_{ij} = \left| \log\left(\frac{I_{cfj}}{I_{cfi}}\right) - \log\left(\exp\left(\frac{V_{bej} + \alpha_1 \frac{\partial V_{be} \Delta T}{\partial T}}{n_{cf} V_0}\right) - 1\right) \right|
\]

\[
FCT_{ij} = \left| \log\left(\frac{V_{bej} + \alpha_1 \frac{\partial V_{be} \Delta T}{\partial T}}{n_{cf} V_0}\right) - \log\left(\exp\left(\frac{V_{bei} + \alpha_1 \frac{\partial V_{be} \Delta T}{\partial T}}{n_{cf} V_0}\right) - 1\right) \right|
\]

with \(I_{cfj} > I_{cfi}\) and \(V_{bej} > V_{bei}\).

A global error function is defined as

\[
FCT = \sum_{ij} FCT_{ij}
\]

The ideality factor \(n_{cf}\) and the parameter \(\alpha_1\) can be determined by minimizing the global error function \(FCT\) for all the possible points \((I_{cfj}, V_{bej})\) and \((I_{cfi}, V_{bei})\) of the forward Gummel measurements.

Once \(\alpha_1\) and \(n_{cf}\) are known, then, the saturation current can be determined form the intercept at the ordinate of the curve \(\log(I_{cf})\) versus \(\log(\exp(V_{beint} + \alpha_1 (\partial V_{be} / \partial T) \Delta T / n_{cf} V_0) - 1)\).

**D. Determination of the forward base current parameters**

The forward base current is considered as the sum of three current components: \(I_{bf1}\), \(I_{bf2}\), and \(I_{bf3}\). The reverse hole-injection, represented by \(I_{bf1}\), is negligible in modern HBT structures. Nevertheless, it is included in our model for sake of completeness, while it is assumed to be temperature-independent. Thus, the following equations can be written:

\[
I_{bf} = I_{bf1} + I_{bf2} + I_{bf3}
\]

\[
I_{bf1} = I_{bf1} \left(\exp\left(\frac{V_{beint}}{n_{bf1} V_0}\right) - 1\right)
\]

\[
I_{bf2} = I_{bf2} \left(\exp\left(\frac{V_{beint} + \alpha_2 \frac{\partial V_{be} \Delta T}{\partial T}}{n_{bf2} V_0}\right) - 1\right)
\]

\[
I_{bf3} = I_{bf3} \left(\exp\left(\frac{V_{beint} + \alpha_3 \frac{\partial V_{be} \Delta T}{\partial T}}{n_{bf3} V_0}\right) - 1\right)
\]

In the forward Gummel characteristic, we can identify three linear regions as shown in Fig. 6. In each region, a single component of base current is dominant and thus can be identified to the total base current. Ideality factors \(n_{bf1}\) and \(n_{bf2}\), saturation currents \(I_{bf1}\), \(I_{bf2}\), and \(I_{bf3}\), and coefficients \(\alpha_2\) and \(\alpha_3\) can be extracted using the technique presented in the previous paragraph.

**Fig.6.** Comparison between measured (o) and simulated (-) forward Gummel characteristics of base current at the ambient temperature 25°C
E. Determination of the Parameters of the Reverse Base and Collector Currents

The reverse collector current is given by:

$$ I_{cr} = I_{scr} \left( \exp \left( \frac{V_{bc\text{int}}}{n_{cr}V_0} \right) - 1 \right) $$  \hspace{1cm} (20)

The reverse base current is given by:

$$ I_{br} = I_{shr} \left( \exp \left( \frac{V_{bc\text{int}}}{n_{br}V_0} \right) - 1 \right) $$  \hspace{1cm} (21)

where

$$ V_{bc\text{int}} = V_{bc} - R_e (I_r + I_c) - R_b I_b $$  \hspace{1cm} (22)

$V_{bc\text{int}}$ and $V_{bc}$ are the internal and external base-collector voltages, respectively.

The parameters of reverse base and collector currents are determined in a similar way as in the case of the forward currents.

IV. IMPLEMENTATION AND VALIDATION OF THE HBT DC MODEL

The extraction procedure was applied to an on-wafer InGaP/GaAs HBT with an emitter area of 2x25 μm² in the common emitter configuration. DC measurements were performed using a probe station monitored by a program elaborated using HP-Vee software. A thermal chuck was used to control temperature. The HBT DC model was implemented in the commercial simulator ADS as a Symbolically Defined Device (SDD). Table I gives the extracted values of the HBT DC model parameters. Figures 7 and 8 show good agreements between measured and simulated DC output characteristics.

V. CONCLUSION

A temperature-dependent HBT DC model was developed and implemented in ADS. A systematic extraction method for the model parameters has been elaborated.

Excellent agreement between measured and modeled data was obtained over a wide range of operating biases. As proof of robustness of the proposed extraction technique, the predicted and measured $I_c-V_{ce}$ characteristics for both constant $I_b$ and constant $V_{be}$ are found to be in excellent agreement.

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Fig. 7. Comparison between measured (o) and simulated (-) $I_c$-$V_{ce}$ characteristics at constant $I_b$ bias.

Fig. 8. Comparison between measured (o) and simulated (-) $I_c$-$V_{ce}$ characteristics with constant $V_{be}$ bias.

REFERENCES


